1 Ultimatum Bargaining

a) The game tree is

![Game Tree Diagram]

b) By backward induction, given $s$, a best response\(^1\) of player 2 is

\[
\begin{cases} 
  \text{Accept} & \text{if } s \leq 1 \\
  \text{Reject} & \text{otherwise}
\end{cases}
\]

Given the best response, player 1 will make the offer $s = 1$, which gives him the highest share. Hence, the subgame perfect equilibrium is $(s = 1, \text{Accept} \forall s \in [0,1])$.

c) Yes, there is this kind of Nash equilibrium. Consider the following strategy profile,

\[
(s = \frac{1}{2}, \begin{cases} 
  \text{Accept} & \text{if } s = \frac{1}{2} \\
  \text{Reject} & \text{otherwise}
\end{cases})
\]

It is a Nash equilibrium. For player 1, he can only get a positive share when $s = \frac{1}{2}$. For player 2, he will accept and get a positive share when $s = \frac{1}{2}$, and feels indifferent between acceptance and rejection when $s \neq \frac{1}{2}$. Hence, given the other’s strategy, everyone’s strategy is his best response.

\(^1\)Note that we pick up a best response that player 2 will accept when $s = 1$. If player 2 will reject when $s = 1$, player 1 will make an offer $s \rightarrow 1$. At that time, there is no SPE.
2 Long Run-Short Run/Repeated Game

a) The best response is

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3, 2*</td>
<td>0, 0</td>
</tr>
<tr>
<td>D</td>
<td>*5, 0</td>
<td><em>1, 1</em></td>
</tr>
</tbody>
</table>

Hence, the static Nash Equilibrium is (D, D).

b) Yes it is a subgame perfect equilibrium outcome path when $\delta = \frac{1}{2}$. Consider the following grim trigger strategy: In the first period, both long-run and short-run player play C. After that, the long-run player and sequential short-run ones play C if the outcome was always (C, C) in every previous period; otherwise, they play D. We then show that given the trigger strategy, everyone’s strategy is his best response in every subgame:

For the long-run player, in the first period and those subgames that the outcome was always (C, C) in every previous period, playing C is a best response since

\[
C : (1 - \delta)[3 + 3\delta + 3\delta^2 + ...] = 3
\]

\[
D : (1 - \delta)[5 + \delta + \delta^2 + ...] = 5 - 4\delta = 3 \leq 3
\]

And in the subgames that someone played D in some previous periods, playing D is a best response since

\[
C : (1 - \delta)[0 + \delta + \delta^2 + ...] = \delta = \frac{1}{2}
\]

\[
D : (1 - \delta)[1 + \delta + \delta^2 + ...] = 1 \geq \frac{1}{2}
\]

For short-run players, in the first period and those subgames that the outcome was always (C, C) in every previous period, playing C is a static best response since $2 \geq 0$. And in the subgames that someone played D in some previous periods, playing D is a static best response since $1 \geq 0$.

From the argument above, we conclude that the trigger strategy is a subgame perfect equilibrium.

c) Yes, there is a subgame perfect equilibrium in which row player’s average payoff is 4. Note that $(4, 1) = \frac{1}{2}(3, 2) + \frac{1}{2}(5, 0)$, and the min max is 1 for both players. Hence, (4, 1) is in the socially feasible individually rational set. By the Folk Theorem, when players are patient enough ($\delta$ is large enough), the average payoff 4 can be supported as a subgame perfect equilibrium.
3 Stackelberg Equilibrium

a) Define \((x_n, x_m)\) are the innovator and imitator’s production levels. By backward induction, given \(x_n\), the imitator’s profit function is:

\[
\pi_m = (20 - x_n - x_m)x_m - 3x_m
\]

F.O.C.

\[
17 - x_n - 2x_m = 0
\]

The best response of the imitator is

\[
x_m = \frac{17 - x_n}{2}
\]

Given the best response, the innovator’s profit function is:

\[
\pi_n = (20 - x_n - \frac{17 - x_n}{2})x_n - 3x_n - R
\]

F.O.C.

\[
\frac{17}{2} - x_n = 0
\]

The best response of the innovator is

\[
x_n = \frac{17}{2}
\]

Hence, the Stackelberg Equilibrium is \((x_n, x_m) = (\frac{17}{2}, \frac{17-x_n}{2})\).

b) The equilibrium outcome is \((\frac{17}{2}, \frac{17}{4})\). Hence, the equilibrium profits of the innovator and imitator are

\[
\pi_n = \frac{289}{8} - R
\]
\[
\pi_m = \frac{289}{16}
\]

Being imitator is preferable to being innovator if

\[
\frac{289}{8} - R \leq \frac{289}{16}
\]
\[
R \geq \frac{289}{16}
\]