PROBLEM SET #5 SOLUTIONS

1. First we need to determine the events in this problem. Let $E$ be the event that evidence is found and let $B$ be the event that vase is broken due to bad packaging. Then, we need to determine $P(B|E)$. Since we are given the following probabilities

\[
P(B) = 0.1
\]
\[
P(E|B) = 0.9
\]
\[
P(E|B^c) = 0.3
\]

by using Baye’s Law, we obtain

\[
P(B|E) = \frac{P(E|B)P(B)}{P(E|B)P(B) + P(E|B^c)P(B^c)}
= \frac{(0.9)(0.1)}{(0.9)(0.1) + (0.3)(0.9)}
= 0.25
\]

2. Expected utility that is gained from Gamble $A$ is calculated as follows;

\[
Eu(A) = P(x = 5)u(5) + P(x = 2)u(2)
= \frac{1}{2}(20 - \frac{20}{5}) + \frac{1}{2}(20 - \frac{20}{2})
= 13
\]

Similarly, expected utility that is gained from Gamble $B$ is

\[
Eu(B) = P(x = 10)u(10) + P(x = 1)u(1)
= \frac{1}{2}(20 - \frac{20}{10}) + \frac{1}{2}(20 - \frac{20}{1})
= 9
\]

Hence, gamble $A$ is chosen.

3. (a) In order to determine pure strategy Nash equilibria (PSNE) of this game, best responses are underlined on the payoff matrix which is given below;
Hence, \((D, R)\) is the unique PSNE.

Since \(L\) is strictly dominated by \(R\) for column player, column player never randomizes between \(L\) and \(R\). Knowing this, it is always optimal to play \(D\) for row player. Hence, there is no mixed strategy Nash equilibria (MSNE).

(b) Similarly, best responses are underlined on the payoff matrix which is given below;

\[
\begin{array}{c|c}
      & L   & R   \\
\hline
U    & 0,0 & 30,60 \\
D    & 90,10 & 20,0 \\
\end{array}
\]

Hence, \((D, L)\) and \((U, R)\) are the PSNE.

Let \(p\) be the probability of playing \(U\) for row player, and then obviously \(1 - p\) be the probability of playing \(D\) (i.e., \(\sigma_1 = (p, 1 - p)\)). So, column player randomizes if the expected payoff of playing \(L\) and \(R\) are the same. Thus,

\[
u_2(\sigma_1, L) = u_2(\sigma_1, R)\\n\begin{align*}
(p)(0) + (1 - p)(10) &= (p)(60) + (1 - p)(0) \\
(1 - p)10 &= 60p \\
p &= 1/7
\end{align*}
\]

Similarly, let \(q\) be the probability of playing \(L\) for column player, then obviously \(1 - q\) be the probability of playing \(R\) (i.e., \(\sigma_2 = (q, 1 - q)\)). So, row player randomizes if the expected payoff of playing \(U\) and \(D\) are the same. Thus,

\[
u_1(U, \sigma_2) = u_1(D, \sigma_2)\\n\begin{align*}
(q)(0) + (1 - q)(30) &= (q)(90) + (1 - q)(20) \\
(1 - q)10 &= 90q \\
q &= 1/10
\end{align*}
\]

Hence, \(((\frac{1}{7}, \frac{6}{7}), (\frac{1}{10}, \frac{9}{10}))\) is the MSNE.
(c) Similarly, best responses are underlined on the payoff matrix which is given below:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>8, 4</td>
<td>6, 10</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>4, 8</td>
<td>8, 4</td>
</tr>
</tbody>
</table>

Hence, there is no PSNE.

Let $p$ be the probability of playing $U$ for row player, and then obviously $1 - p$ be the probability of playing $D$ (i.e., $\sigma_1 = (p, 1 - p)$). So, column player randomizes if the expected payoff of playing $L$ and $R$ are the same. Thus,

$$u_2(\sigma_1, L) = u_2(\sigma_1, R)$$

$$(p)(4) + (1 - p)(8) = (p)(10) + (1 - p)(4)$$

$$(1 - p)4 = 6p$$

$$p = \frac{2}{5}$$

Similarly, let $q$ be the probability of playing $L$ for column player, then obviously $1 - q$ be the probability of playing $R$ (i.e., $\sigma_2 = (q, 1 - q)$). So, row player randomizes if the expected payoff of playing $U$ and $D$ are the same. Thus,

$$u_1(U, \sigma_2) = u_1(D, \sigma_2)$$

$$(q)(8) + (1 - q)(6) = (q)(4) + (1 - q)(8)$$

$$4q = (1 - q)2$$

$$q = \frac{1}{3}$$

Hence, $((\frac{2}{5}, 3), (\frac{1}{3}, \frac{2}{3}))$ is the MSNE.