Introductory Lecture

*What this class is about*

- economic science as it exists today
- intrinsically a mathematical subject
- the heart of economics is mechanism design theory
- the goal of the class is to give you a limited working knowledge of mechanism design theory
The Monopoly Pricing Problem

- The catering company Big Eats has the exclusive right to sell pizza on the campus of Big U.
- How much should it charge for each pizza?
- Each pizza will cost \( c \) to produce and distribute.
- Market research indicates that the number of units that will be sold \( x \) depends upon the price \( p \) according to the relation \( x = d(p) \), where a higher price results in fewer sales.
- This is the simplest example of a mechanism design problem: here the choice is between different prices that can be charged. Deeper analysis would consider more elaborate pricing schemes: auction the pizzas to the highest bidder, allocate the pizzas by means of a contest and so forth.
- Illustrates the interplay between an economic problem (what should we do with the pizzas?) and mathematical methods.
Solution to the Problem of Monopoly

$p$ is price, $x$ is output, $c$ is unit cost

profit $\pi = px - cx$; this is what Big Eats cares about

demand $x = d(p)$ or inverse demand $p = f(x)$

profit again $\pi = f(x)x - cx$

for a maximum: marginal profit equals zero

$$\frac{d\pi}{dx} = f'(x)x + f(x) - c = 0, \quad f(x)\left[\frac{f'(x)x}{f(x)} + 1\right] = c$$

$$\eta \equiv \frac{d \log x}{d \log p} = \frac{d \log x}{d \log f(x)} = \frac{1/x}{f'(x)/f(x)} = \frac{f(x)}{f'(x)x}$$ the price elasticity of demand

$$p\left[\frac{1}{\eta} + 1\right] = c \text{ or } p - c = -p/\eta$$
Discussion of the Solution

\[ p - c = -\frac{p}{\eta} \]

\( \eta \) is negative so the markup \( p - c \) is positive

- monopoly vs. “competition”: the more “elastic” is output [large absolute \( \eta \)] with respect to price the smaller the markup
- competition: raise price a tiny amount lose entire market: infinite elasticity
- the more “inelastic” is output [small absolute \( \eta \)] with respect to price, the bigger the markup: monopolists like inelasticity, you can increase your price a lot without having much effect on your sales
- game theoretic perspective: we are taking into account how “other players” respond to our “strategy”: the more we charge, the less the “other players” are willing to pay
An Example with Linear Demand

\[ p = a - bx \]

monopoly

\[ \pi = (a - bx)x - cx = (a - c)x - bx^2 \]

\[ \frac{d\pi}{dx} = (a - c) - 2bx = 0 \]

\[ x = \frac{a - c}{2b} \] the monopoly output

competitive equilibrium

\[ p = c \]

\[ a - bx = c \]

\[ x = \frac{a - c}{b} \] twice the monopoly output
Graphical Analysis

revenue = \( px = f(x)x \)

marginal revenue = \( MR = \frac{d}{dx} \text{revenue} \)

cost = \( cx \)

marginal cost = \( MC = \frac{d}{dx} \text{cost} = c \)

\( f'(x)x + f(x) = c \) or \( MR = MC \)

take a=9, b=1, c=2
Optimum of the Monopolist

Inverse demand
MC
MR

Output
Returns to Scale

total cost = \( cx + dx^2 / 2 \)
average = \( c + dx / 2 \)
marginal = \( c + dx \)

- if \( d = 0 \) constant returns to scale
- if \( d > 0 \) decreasing returns to scale
- if \( d < 0 \) increasing returns to scale
Example Revisited

\[ p = a - bx \]

monopoly

\[ \pi = (a - bx)x - cx - dx^2 / 2 \]

\[ = (a - c)x - (b + d / 2)x^2 \]

\[ \frac{d\pi}{dx} = (a - c) - 2(b + d / 2)x = 0 \]

\[ x = \frac{a - c}{2b + d} \]
competitive equilibrium

\[ a - bx = c + dx \]

\[ x = \frac{a - c}{b + d} \]

- when \( d > 0 \) (decreasing returns to scale) monopolist produces more than \( \frac{1}{2} \) competition
- when \( d < 0 \) competitor earns negative profit

average = \( c + dx / 2 \)

marginal = \( c + dx \)

when \( d < 0 \)

average cost > marginal cost

so price = marginal cost < average cost

means you lose money on each unit you sell