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The Repeated Prisoner’s Dilemma

recall the prisoner’s dilemma game

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
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<tbody>
<tr>
<td>Player 1</td>
<td>don’t confess</td>
</tr>
<tr>
<td>don’t confess</td>
<td>32,32</td>
</tr>
<tr>
<td>confess</td>
<td>35,28</td>
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</tbody>
</table>

- This is a simultaneous move game with a unique Nash equilibrium, and a unique strictly dominant strategy solution at 30, 30.
- The unique non-cooperative solution is Pareto dominated by 32, 32
- with repeated play, incentive are changed by the possibility of punishments and rewards in the future.
Intertemporal Preference

$u_i(t)$ is the utility or payoff received by player $i$ in period $t$

the game is repeated indefinitely and that intertemporal preferences
are given by average present value

$$U_i = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_i(t)$$

where the common discount factor $\delta$ is between 0 and 1.

- a basic feature of repeated games: regardless of the discount factors,
  the repeated static equilibrium is a subgame perfect equilibrium of
  the repeated game
**Grim Strategies**

the *grim strategy* in the repeated game is

- cooperate in the first period
- cooperate in subsequent periods as long as all players have cooperated in every previous period
- cheat in any period in which some player has cheated in any previous period
suppose the other player plays the grim strategy

• payoff to cheating

\[(1 - \delta)(35 + 30\delta + 30\delta^2 \ldots) = (1 - \delta)35 + 30\delta = 35 - 5\delta\]

• payoff to cooperating

32

• optimal to cooperate if

\[32 \geq 35 - 5\delta\] or

\[\delta \geq \frac{3}{5}\]

• if \(\delta \geq \frac{3}{5}\) both players playing the grim strategy is a subgame perfect equilibrium
why subgame perfect?