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Monopoly

Big Media Giant (BMG), the gigantic media company has a new album by the group Lucky in the Park.

What price should it charge for this new product?

Each unit will cost $c to produce and distribute.

Market research indicates that the number of units that will be sold $x$ depends upon the price $p$ according to the relation $x = d(p)$
Monopoly Solution

p is price, x is output, c is unit cost

profit \( \pi = px - cx \)

inverse demand \( x = d(p) \) or \( p = f(x) \)

profit again \( \pi = f(x)x - cx \)

marginal profit equals zero

\[
\frac{d\pi}{dx} = f'(x)x + f(x) - c = 0, \quad f(x) \left[ \frac{f'(x)x}{f(x)} + 1 \right] = c
\]

\[
p \left[ \frac{d \log p}{d \log x} + 1 \right] = c
\]
Discussion of the Solution

\[ p \left[ \frac{d \log p}{d \log x} + 1 \right] = c \]

\[ \frac{d \log p}{d \log x} \] is negative so \( p > c \)

- monopoly vs. “competition”
- the more “inelastic” is price with respect to output, the bigger the markup
- take into account how other “players” respond to your “strategy”: the more you sell, the lower the price “opponents” are willing to pay
**An Example with Linear Demand**

\[ p = a - bx \]

monopoly

\[ \pi = (a - bx)x - cx = (a - c)x - bx^2 \]

\[ \frac{d\pi}{dx} = (a - c) - 2bx = 0 \]

\[ x = \frac{a - c}{2b} \]

competitive equilibrium

\[ p = c \]

\[ a - bx = c \]

\[ x = \frac{a - c}{b} \]
Graphical Analysis

revenue = px = f(x)x

marginal revenue = MR = \frac{d}{dx} revenue

cost = cx

marginal cost = MC = \frac{d}{dx} cost = c

f'(x)x + f(x) = c or MR = MC

take a=9, b=1, c=2
Optimum of the Monopolist

Output

- inverse demand
- MC
- MR
**Returns to Scale**

\[
\text{total cost} = cx + dx^2 / 2
\]

\[
\text{average} = c + dx / 2
\]

\[
\text{marginal} = c + dx
\]

- if \( d = 0 \) constant returns to scale
- if \( d > 0 \) decreasing returns to scale
- if \( d < 0 \) increasing returns to scale


**Example Revisited**

\[ p = a - bx \]

monopoly

\[ \pi = (a - bx)x - cx - dx^2 / 2 \]

\[ = (a - c)x - (b + d / 2)x^2 \]

\[ \frac{d\pi}{dx} = (a - c) - 2(b + d / 2)x = 0 \]

\[ x = \frac{a - c}{2b + d} \]
competitive equilibrium

\[ a - bx = c + dx \]

\[ x = \frac{a - c}{b + d} \]

- when \( d > 0 \) (decreasing returns to scale) monopolist produces more than \( \frac{1}{2} \) competition

- when \( d < 0 \) competitor earns negative profit

\[ \text{average} = \frac{c + dx}{2} \]

\[ \text{marginal} = c + dx \]

when \( d < 0 \)

average cost > marginal cost

so price = marginal cost < average cost

means you lose money on each unit you sell