Types and Incomplete Information Cournot Competition

- What happens when players do not know one another’s payoffs?
- Games of “incomplete information” versus games of “imperfect information”
- Harsanyi’s notion of “types” encapsulating “private information”
- Nature moves first and assigns each player a type; player’s know their own types but not their opponents’ types
- Players do have a common prior belief about opponents’ types

Bayesian Games

There are a finite number of types $\theta_i \in \Theta_i$

There is a common prior $p(\theta)$ shared by all players

$p(\theta_{-i}|\theta_i)$ is the conditional probability a player places on opponents' types given his own type

The *stage* game has finite strategy spaces $s_i \in S_i$ and has utility functions $u^i(s,\theta)$

Bayesian Equilibrium

A *Bayesian Equilibrium* is a Nash equilibrium of the game in which the strategies are maps from types Θ_i to stage game strategies S_i

This is equivalent to each player having a strategy as a function of his type $s_i(\theta_i)$ that maximizes conditional on his own type θ_i (for each type that has positive probability)

$$\max_{s_i} \sum_{\theta_{-i}} u_i(s_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) p(\theta_{-i} | \theta_i)$$

The Cournot Model with Types

- A duopoly with demand given by $p = 17 - x$
- A firm's type is its cost, known only to that firm: each firm has a 50-50 chance of cost constant marginal cost 1 or 3.

profits of a representative firm

$$\begin{aligned}\pi_i(c_i, x) &= \left[17 - (x_i + x_{-i})\right]x_i - c_i x_i \\ &= \left[17 - c_i - (x_i + x_{-i})\right]x_i\end{aligned}$$

Let us look for the symmetric pure strategy equilibrium

Finding the Bayes-Nash Equilibrium

x^1, x^3 will be the output chosen in response to cost

$$\pi_i(x_i, c_i) = .5\left[17 - c_i - (x_i + x^1)\right]x_i$$
$$+.5\left[17 - c_i - (x_i + x^3)\right]x_i$$
$$= \left[17 - c_i - (x_i + .5x^1 + .5x^3)\right]x_i$$

maximize with respect to x_i

$$\frac{d\pi_i(x_i, c_i)}{dx_i} = \left[17 - c_i - (x_i + .5x^1 + .5x^3)\right] - x_i$$
$$= \left[17 - c_i - (2x_i + .5x^1 + .5x^3)\right] = 0$$

so $x_i = (17 - c_i - .5x^1 - .5x^3) / 2$

$$x_i = (17 - c_i - .5x^1 - .5x^3) / 2$$

$$x^1 = (16 - .5x^1 - .5x^3) / 2$$

$$x^3 = (14 - .5x^1 - .5x^3) / 2$$

add the two equations together

$x^1 + x^3 = 15 - .5(x^1 + x^3)$, or $x^1 + x^3 = 10$

substituting back in we have $x^1 = 11/2$, $x^3 = 9/2$

industry output

probability ¼ 11

probability ½ 10

probability ¼ 9

Suppose by contrast costs are known

If both costs are 1 then competitive output is 16 and Cournot output is 2/3rds this amount 10 2/3

If both costs are 3 then competitive output is 14 and Cournot output is 9 1/3

If one cost is 1 and one cost is 3

$$x^1 = 8 - x^3/2, \; x^3 = 7 - x^1/2$$

which gives $x^1 + x^3 = 10$

With known costs, mean industry output is the same as with private costs, but there is less variation in output