### 1. Short Answers

For each of the normal form games below, find all of the Nash equilibria. Which are Pareto Efficient?

#### a)

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1,0</td>
<td>3*,1*(Efficient)</td>
</tr>
<tr>
<td>D</td>
<td>2*,2*(Efficient)</td>
<td>1,0</td>
</tr>
</tbody>
</table>

Nash Equilibria:

- (D,L): Efficient
- (U,R): Efficient

#### b)

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>2*,5* (Not efficient)</td>
<td>6*,3</td>
</tr>
<tr>
<td>D</td>
<td>0,9*</td>
<td>4,7</td>
</tr>
</tbody>
</table>

Nash Equilibrium:

- (U,L): Not efficient

For each of the extensive form games below, find the normal form and all Nash equilibria. Then find all of the subgame perfect equilibria. Which are Pareto Efficient?

#### c)

Extensive form with subgame perfect choices marked with dashed lines
normal form with best response correspondence and Nash equilibria marked

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>100*,100* (Efficient)</td>
<td>100,100*</td>
</tr>
<tr>
<td>D</td>
<td>1,1</td>
<td>101*,2* (Efficient)</td>
</tr>
</tbody>
</table>

Nash equilibria / Subgame Perfect equilibria:

(U,u): Efficient
(D,d): Efficient

d) Extensive form with subgame perfect choices marked with dashed lines

Subgame perfect equilibria:

(d, U): Not efficient
(d, D): Efficient

normal form with best response correspondence and Nash equilibria marked

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>D</th>
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</thead>
<tbody>
<tr>
<td>u</td>
<td>0*,0</td>
<td>100,100*</td>
</tr>
<tr>
<td>d</td>
<td>0*,0* (Not efficient)</td>
<td>101*,0* (Efficient)</td>
</tr>
</tbody>
</table>

Nash equilibria:

(d, U): Not efficient
(d, D): Efficient
2. Demand function: \( P = 17 + (x_1 + x_2) \)
   \( MC_1 = 1 \)
   \( MC_2 = 3 \)

(a)
\[
\begin{align*}
\text{Profit function for firm 1:} & \quad \pi_1 = P x_1 - \frac{1}{2} x_1 \left[ (17 + x_1 + x_2) x_1 \right] \quad \text{firm 1 reaction function} \\
\text{Profit function for firm 2:} & \quad \pi_2 = P x_2 - \frac{3}{2} x_2 \left[ (17 + x_1 + x_2) x_2 \right] \quad \text{firm 2 reaction function}
\end{align*}
\]

(b)
\[
\begin{align*}
\frac{\partial \pi_1}{\partial x_1} = 17 - 2x_1 & \quad \Rightarrow x_1 = \frac{16 - x_2}{2} \\
\frac{\partial \pi_2}{\partial x_2} = 17 - 3x_2 & \quad \Rightarrow x_2 = \frac{14 - x_1}{2}
\end{align*}
\]

Combining both reaction functions:
Nash Equilibrium: \( x_1 = 6, \quad x_2 = 4, \quad P = 7 \)

(c)
\[
\begin{align*}
\pi_1 = \begin{cases} 
8 & \text{if } P_1 > P_2 \\
\frac{17 P_1 - x_1}{2} & \text{if } P_1 = P_2 \\
\frac{17 P_1 - x_1}{2} & \text{if } P_1 < P_2 \\
\frac{17 P_2 - x_2}{2} & \text{if } P_2 < P_1 \\
\frac{17 P_2 - x_2}{2} & \text{if } P_2 = P_1
\end{cases}
\]

(d) Assume at \( P = 3 \) firm 2 decides not to produce. Then, the Nash Equilibrium is: \( P^* = 3, \quad X^* = 14 \)

e) More output is produced under Bertrand competition.
3. How to sell a car:

The three players in this game and the actions they can take are the following:

- You – you value the car at $0
  - $S$: sell the car in a second price sealed bid auction
  - $Ph$: sell the car by setting a take-or-leave-it price of $3400.
  - $Pl$: sell the car by setting a take-or-leave-it price of $2400.

- Buyer 1 – values the car at $3500
  - $H(L)$: bid $3400 ($2400) in the 2nd price auction
  - $t(l)$: take (leave) the set price in the take-it-or-leave-it scheme

- Buyer 2 – values the car at $2500
  - $H(L)$: bid $3400 ($2400) in the 2nd price auction
  - $t(l)$: take (leave) the set price in the take-it-or-leave-it scheme

a) The extensive form. Note that the payoffs are the triples of (you,buyer1,buyer2).
b) The subgame perfect equilibrium of this three player game is

\[ SPE : (P_h, Htt, LHlt) \]

as illustrated by the thick lines in the extensive form.

- \( P_h \) is your strategy –
  - \( P_h \): set a take-it-or-leave-it price of $3400.

- \( Htt \) is buyer 1’s strategy –
  - \( H \): bid $3400 if you sell the car in a second price auction; \( t \): take if you set the take-or-leave-it price of $3400; \( t \): take if you set the take-or-leave-it price of $2400.

- \( LHlt \) is buyer 2’s strategy –
  - \( L \): bid $2400 if you sell the car in a second price auction and buyer 1 bid $3400; \( H \): bid $3400 if you sell the car in a second price auction and buyer 1 bid $2400; \( l \): leave if you set the take-or-leave-it price of $3400 (and buyer 1 leaves); \( t \): take if you set the take-or-leave-it price of $2400 (and buyer 1 leaves).