Final Exam Answers: Economics 101

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1. Normal Form Games (note that a complete answer must include a drawing of the socially feasible sets)

a)

	L	R
U	-1*,0	2,1*
D	-3,3*	4*,2

No pure strategy equilibrium. Unique mixed equilibrium where both players mix 50-50. Utility is (½ , 1 ½) is Pareto dominated by 4,2 so not Pareto Efficient. No weakly dominated strategies. Pure strategy maxmin for player 1 is –1, for player 2 is 1; pure strategy minmax for player 1 is –1, for player 2 is 1.

b)

	L(q)	R(1-q)
U(p)	-1*,5*	-1,1
D(1-p)	-2,1	5*,4*

Two pure strategy equilibria as marked. Mixed for player 2 $-1 = -2q + 5(1-q)$ so q=6/7; for player 1 $5p + (1-p) = p + 4(1-p), 7p = 3$ so p=3/7. Pure strategy equilibria are Pareto Efficient. The mixed equilibrium has payoffs of (-1, 2 5/7) is not. No weakly dominated strategies. Pure strategy maxmin for player 1 is –1 for player 2 is 1; pure strategy minmax for player 1 is –1 for player 2 is 4.

c)

	L	R
U	5,5	9,8*
D	8*,9	11*,10*

Unique Nash equilibrium (U and L are both strictly dominated). No mixed equilibria due to dominance. Nash equilibrium is Pareto efficient. Pure maxmin for player 1 is 8 for player 2 is 8. Pure minmax for player 1 is 8, for player 2 is 8.

2. Repeated Games

	L	R
U	4,4	0,8
D	5,0	1,1 (static Nash)

Use the grim strategies: U(or L) as long as UL in every past period, otherwise DR (the static Nash equilibrium). Player 2 has the greatest gain to deviating (4). If he deviates he gets at most $(1-\delta)8+\delta 1 \le 4$ or $4 \le 7\delta$, so this is an equilibrium for $\delta \ge 4/7$. This is subgame perfect since the punishment is a static Nash equilibrium. Minmax here is 1 for both players, so Folk Theorem says for δ close enough to 1 that SFIR region exceeding (1,1) are all subgame perfect equilibria.

3. Long Run versus Short Run

	L	R
U	2,4*	0,0
M	5,0	1*,11*
D	11*,0	1*,3*

There are two static Nash equilibrium is MR and DR; the Stackelberg equilibrium is UL. Strategies for which lead to playing UL are UL if always UL in the past and DR if ever a deviation. Alternatively, players may base their strategies on past play of the LR player only: LR: U if U in the past and D if ever a deviation by LR and SR: L if U in the past and R if ever a deviation of the LR player.

These are optimal for the short-run player because it is in his best-response correspondence. For the long run player it must be that $2 \geq (1-\delta)11 + \delta 1$ or $\delta \geq 9/10$.

4. Decision Analysis

An employer must decide whether or not to introduce mandatory drug testing for his employees. The test is correctly identifies a drug user 90% of the time. It mistakenly identifies a non-drug user as a drug user 5% of the time. Employing a drug using employee costs $10,000 per year. Firing a non-drug using employee costs $20,000 per year. In this industry, 25% of employees use drugs. How much is the employer willing to pay (per year) for the testing program?

No testing program: fire all employees gives .25x0 - .75x 20 = -15; fire no employees gives -.25x10+.75x0= -2.5. So keep all employees and get –2.5.

Testing program: This can only be worthwhile if we fire when positive and don't fire when negative. Use Bayes law we have

$$pr(drug|+) = \frac{pr(+|drug)\,pr(drug)}{pr(+|drug)\,pr(drug) + pr(+|nodrug)\,pr(nodrug)} = \frac{.9x.25}{.9x.25+.05x.75} = .857$$

note that $pr(+) = pr(+|drug)\,pr(drug) + pr(+|nodrug)\,pr(nodrug) = .9x.25+.05x.75 = .26$

$$pr(drug|-) = \frac{pr(-|drug)\,pr(drug)}{pr(-|drug)\,pr(drug) + pr(-|nodrug)\,pr(nodrug)} = \frac{.1x.25}{.1x.25+.95x.75} = .034$$

Hence utility with testing program is

.26x(.857x0+.143x(-20))+.74(-.034x10+.966x0) ≈ -1

So you will pay roughly $1,500 per year for the program

5. Cournot with Uncertain Cost

Consider a Cournot Duopoly with demand $p = 17 - x$. There are two possible levels of marginal cost: low and equal to 1 or high and equal to 3. There is a 40% chance both firms are high cost, a 40% chance they are both low cost, a 10% chance firm 1 is high cost and firm 2 low cost, and a 10% chance firm 1 is low cost and firm 2 high cost. Assuming that each firm knows its own marginal costand these probabilities, in the Bayesian Nash equilibrium of the Cournot game, what are the equilibrium strategies of the two firms?

	1	3
1	.4	.1
3	.1	.4

Let x^1 be equilibrium output of low cost firm, x^3 of high cost firm

$pr(1|1) = pr(1,1)/pr(1) = .4/.5 = .8$

$pr(1|3) = pr(1,3)/pr(3) = .1/.5 = .2$

Profit given low cost: $.8(16-(x^1+x_i))x_i + .2(16-(x^3+x_i))x_i = (16-x_i - .8x^1 - .2x^3)x_i$

FOC is $16-2x_i - .8x^1 - .2x^3 = 0$ so $16-2x^1 - .8x^1 - .2x^3 = 0$ or $16 = 2.8x^1 + .2x^3$

Profit given high cost: $.2(14-(x^1+x_i))x_i + .8(14-(x^3+x_i))x_i = (14-x_i - .2x^1 - .8x^3)x_i$

FOC is $14-2x_i - .2x^1 - .8x^3 = 0$ so $14-2x^3 - .2x^1 - .8x^3 = 0$ or $14 = 2.8x^3 + .2x^1$ or $196 = 39.2x^3 + 2.8x^1$

Subtracting these two equations give $180 = 39x^3$ or $x^3 = 4\,{}^{8}\!/_{13}$

Plugging back in gives $x^1 = (16 - .2 * {}^{60}\!/_{13})/2.8 = 5.38$

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