Damned If You Do and Damned If You Don't: Two Masters[☆]

Rohan Dutta[1], David K. Levine[2], Salvatore Modica[3]

Abstract

We study common agency problems in which principals (groups) make costly commitments to incentives that are conditioned on imperfect signals of the agent's action. Our framework allows for incentives to be either rewards or punishments and an equilibrium always exists. For our canonical example with two principals we obtain a unique equilibrium, which typically involves randomization by both principals. Greater similarity between principals leads to more aggressive competition. The principals weakly prefer punishment to rewards, sometimes strictly. With rewards an agent voluntarily joins both groups; with punishment it depends on whether severe punishments are feasible and cheap for the principals. We study whether introducing an attractive compromise reduces competition between principals. Our framework of imperfect monitoring offers a natural perturbation of the standard common agency model, which results in sharper equilibrium predictions. The limit equilibrium prediction provides support to both truthful equilibria and the competing notion of natural equilibria, which unlike the former may be inefficient.

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[*]Corresponding author David K. Levine, 1 Brooking Dr., St. Louis, MO, USA 63130

Email addresses: `rohan.dutta@mcgill.ca` (Rohan Dutta), `david@dklevine.com` (David K. Levine), `salvatore.modica@unipa.it` (Salvatore Modica)

[1]Department of Economics, McGill University

[2]Department of Economics, EUI and WUSTL

[3]Università di Palermo

No one can serve two masters, for either he will hate the one and love the other, or he will be devoted to the one and despise the other. *Matthew 6:24*.

1. Introduction

We study common agency problems where the agent may be rewarded or punished for what he does, and the level of punishment or reward is determined by the principals' effort. Effort, and therefore both rewards and punishments, is costly to the principals. A prototypical situation we have in mind is that of two groups competing for the indivisible effort of a member of both groups: for example to attend a rally for or against a referendum, or to vote with one party or the other. For example, you might be Catholic and a Democrat, and the Catholics say vote for the anti-abortion candidate and the Democrats say vote for the pro-abortion candidate. What do you do? How do the groups optimally provide incentives in the presence of a competing group? Typically social groups provide incentives not only through costly rewards but also through costly punishments. For instance, you may value your social connections with both groups, and both can threaten to deny you the benefits of belonging if you do not do as they say. The existing literature focuses on costly rewards, and when punishments take place they typically are not costly to the principal but instead constitute a transfer from the agent to the principal.

The other key component of our analysis is imperfect monitoring of the agent's action. Besides the practical fact that noise is always present this creates punishment costs on the equilibrium path and avoids the degenerate possibility that principals pile on punishment that will never take place. In our setup principals commit to incentives that are conditioned on imperfect signals of the agent's action. Combined with costly effort this means that the "losing" principal pays something and typically implies that equilibria must involve mixed strategies. This is in contrast to the standard common agency model which focuses on pure strategy equilibria.

Our canonical example examines the case of two principals with the agent choosing one of three actions. One of these constitutes inaction, the other two favor one or the other principal. In sharp contrast to the common agency literature, the equilibrium prediction under all parameter configurations is unique. For parameter values that lead to both principals actively competing, the equilibrium is in mixed strategies. Irrespective of whether effort translates into reward or punishment, making the principals more similar leads to more aggressive competition. The principal with the lower willingness to pay, dubbed disadvantaged, does not care whether incentives are provided in the form of punishments or rewards. By contrast the advantaged principal never prefers rewards to punishment and sometimes strictly prefers punishment. The welfare implication for the agent is more subtle: we focus on analyzing which groups the agent would choose to join. With rewards he would join both. With punishments, it depends on a variety of factors such as the signal technology, the principals' valuations and the intensity with which effort gets translated to punishment. The outcomes span the entire spectrum from joining both to none to, in specific scenarios, only the advantaged principal but also, more surprisingly at first sight, only the disadvantaged principal. In particular, we find that if punishments are limited to exclusion from the benefits of joining a principal then both groups are joined, while if severe punishments are possible at low cost we are in the biblical case described above - the competition between the two principals to punish the agent is so ruinous that it never pays to join both.

We then study two variations on the canonical example. First we ask whether compromise by the agent can mitigate the severity of competition between the principals. In the canonical example described above there is a compromise action (inaction) but the principals can only observe whether the agent took their favored action or not and in the latter case they cannot tell if the action taken was inaction or favored the other principal. Further, inaction brings the principals no payoff. As a result such a compromise is not observed in equilibrium. Hence we ask: with a more favorable signal structure for detecting that the agent compromised and better payoffs will the principals induce compromise? If the signal is strong enough we show that this is indeed the case.

Lastly, we investigate how the canonical example relates to the standard common agency literature. Our assumption of imperfect monitoring allows a natural perturbation of complete information common agency (rewards-only) games and a game theoretic formalization of the Bernheim and Whinston (1986a) idea of "serious" strategies.[4] This results in a sharp equilibrium selection and a strategic foundation for truthful equilibria. Interestingly, it also provides support and a strategic foundation to a competing class of equilibria, natural equilibria, introduced by Kirchsteiger and Prat (2001), which unlike truthful equilibria may be inefficient. This makes the case that such inefficiencies may be fundamental to common agency games such as lobbying. This is in contrast to the extensive theoretical as well as empirical literature that takes the efficiency of complete information lobbying for granted due to their reliance on truthful strategies.[5]

2. The Model

We consider a common agency problem. There is one agent who chooses actions from a finite set $a \in A$. There are principals $k = 1, 2, \ldots, K$. Each principal has access to a finite

[4] The relationship between "serious" strategies, truthful equilibria and our perturbation is discussed in detail in section 5

[5] See, for instance, Persson (1998), Grossman and Helpman (1994), Dixit, Grossman and Helpman (1997), Rama and Tabellini (1998), Goldberg and Maggi (1999) and Marceau and Smart (2003) .

set of random signals $z^k \in Z^k$ about the agent's action. The signal profile $z = (z^1, \ldots, z^K)$ is distributed according to $\pi(z|a)$. Each principal makes an effort choice $\phi^k \in \Phi^k$ a compact convex subset of Euclidean space.[6]

The agent has a cost of taking actions $c(a) \geq 0$ where for some $a \in A$ the cost $c(a) = 0$. The agent also receives a scalar punishment from each principal based on the signals and efforts chosen by that principal. This is given by a continuous function $P^k(\phi^k, z^k)$. These punishments may be negative, representing rewards. The overall utility of the agent is then $U(a, \phi) = -c(a) - \sum_{z \in Z} \sum_k \pi(z|a) P^k(\phi^k, z^k)$. Each principal receives a utility $v^k(a)$ from the agents action and pays a cost given by a continuous function $D^k(\phi^k, z^k) \geq 0$ based on the signals and effort chosen. The overall utility of the principal is then $V^k(a, \phi^k) = v^k(a) - \sum_{z \in Z} \pi(z|a) D^k(\phi^k, z^k)$

Play is sequential. In the initial period principals move simultaneously and choose mixed *strategies* F^k that are measures over choices of effort (pure strategies). Their realizations $\phi^k, k = 1, \ldots, K$ are observed by the agent, who then chooses an action $a \in A$. In the third and final period principals receive the noisy signals z^k about how agent played and utility is determined accordingly.

An *optimal response* for the agent is a probability distribution $\alpha(\phi)$ over A that is measurable as a function of the strategy profile ϕ and such that $\alpha(\phi)[a] > 0$ implies $a \in \arg\max_b U(b, \phi)$.

A subgame perfect equilibrium is an optimal response for the agent α and a strategy F^k for each principal that is optimal given α. We refer to this game as the *basic game.*

2.1. Existence and Continuity of Equilibrium

Since agent utility $U(a, \phi)$ and principal utility $V^k(a, \phi^k)$ are continuous in a, ϕ, π we get the following upper hemicontinuity result.

Lemma 1. *Suppose $\phi_n \to \phi$, $\alpha_n \to \alpha$ and that $\pi^n \to \pi$ are such that α_n is an optimal response at n. Then*

1. *α is optimal with respect to ϕ, π*
2. *$V^k(\phi_n, \alpha_n) \to V^k(\phi, \alpha)$*

Proof. (1) is due to the maximum theorem, which ensures that the set of optimal responses is upper hemicontinuous. (2) then follows immediately due to continuity. □

Theorem 1. *An equilibrium of the basic game exists. If $\pi^n \to \pi$ and F_n^k are a sequence of equilibrium distributions for the principals for n that converge weakly to F^k then there*

[6] Note that since the effort space can be multidimensional this formalism allows us to study the case where the principal chooses a signal dependent schedule of efforts.

is an equilibrium of the game with respect to π in which the equilibrium distribution for the principals is F and in which the equilibrium utilities converge.

Notice that there is no assertion about convergence of the strategies for the agent.

Proof. First we show that the *utility* correspondence $V^k(\phi)$ which assigns each ϕ the set of vectors of principals' utility corresponding to some optimal agent play is upper hemi-continuous. Consider a sequence $\phi_m \to \phi$ and $x_m \to x$ with $x_m \in V(\phi_m)$. We need to show that $x \in V(\phi)$. Let α_m be such that $x_m = V(\phi_m, \alpha_m)$. Since the set of agent mixed strategies is compact, the sequence α_m must have a convergent subsequence, say α_n, with $\alpha_n \to \alpha$. Recall that α_n is an optimal response at n. So we have the subsequence ϕ_n, x_n where $x_n = V^k(\phi_n, \alpha_n)$. Then by Lemma 1 we have $\lim x_n = V^k(\phi, \alpha) \in V(\phi)$. Finally note that since $x_m \to x$ it must be that $x = \lim x_n$, and so $x \in V(\phi)$, as required.

$V^k(\phi)$ is obviously bounded and convex valued. Hence by Simon and Zame (1990) an equilibrium exists. We can leverage their corollary that sequences of equilibria converge to equilibria to prove that the equilibrium correspondence is upper hemicontinuous with respect to the signal distribution π. We do that by introducing an artificial additional principal called nature who chooses π and is completely indifferent. □

The generality of this existence result is very helpful for our analysis. Even under parameter configurations which make explicit construction of equilibrium strategies difficult, it preserves the possibility of characterizing and discussing the properties of such equilibria, sure of their existence. Nevertheless, the model is far too general for our purposes. For a sharper picture of the implications of competing groups acting as principals with a common agent we focus henceforth on the special case of two principals with conflicting interests on the agent's actions.

3. Canonical Example

There are two principals $k = 1, 2$. The agent can take three actions $A = \{0, 1, 2\}$. Action 0 represents inaction and is worth $0 = v^k(0) = c(0)$ to both principals and to the agent. For $k \in \{1, 2\}$, we define $-k$ as the element of $\{1, 2\}$ such that $-k \neq k$. The action $k \neq 0$ is worth $v^k(k) = \nu^k > 0$ to principal k and action $-k$ is worth $v^k(-k) = 0$ so that principal k prefers the action k. Each principal k chooses effort $\phi^k \in [0, \Phi^k]$. We assume that $\nu^1 > \nu^2$ and $\Phi^1 > \Phi^2$ and call principal 1 the *advantaged principal*. The action $k \neq 0$ costs $c(k) = c$ to the agent.

Our basic assumption is signal symmetry between the two principals and that each principal can tell whether his preferred action is taken, but if it is not cannot tell what other action the agent might have taken. Specifically, each principal receives an independently drawn signal $z^k \in \{0, 1\}$ where the probability of receiving the "bad" signal 1 when the agent chooses the action k is $\pi(z^k = 1|k) = \pi > 0$ and the probability of receiving the signal 1 when the agent chooses action $j \neq k$ is $1 > \pi(z^k = 1| - k) = \pi(z^k = 1|0) = \pi_b > \pi$.

We consider two cases: in the *punishment case* principal k's effort results in a punishment to the agent of $P^k(\phi^k, 1) = \lambda\phi^k$ when the bad signal 1 is received and $P^k(\phi^k, 0) = 0$ when the good signal 0 is received; in the *reward case* ϕ^k results in a reward of $-P^k(\phi^k, 0) = \lambda\phi^k$ when the good signal 0 is received and of $P^k(\phi^k, 1) = 0$ when the bad signal 1 is received. The term $\lambda > 0$ captures the intensity with which effort gets converted into reward or punishment.

We may use $\mu \in \{0, 1\}$ as an indicator of the punishment case with $\mu = 1$ meaning the punishment case so that on the good signal 0 there is a reward of $(1 - \mu)\lambda\phi^k$ and on the bad signal 1 a punishment of $\mu\lambda\phi^k$ is issued.[7] hence the expected effort cost to principal k if the agent takes action a is given by

$$\left[\mu\pi(z^k = 1|a) + (1 - \mu)\pi(z^k = 0|a)\right] \phi^k.$$

Consequently the overall expected utility of principal k if the agent takes action k is $\nu^k - [(1 - \pi)(1 - \mu) + \pi\mu]\phi^k$; for actions $-k$ and 0 it is $-[(1 - \pi_b)(1 - \mu) + \pi_b\mu]\phi^k$. The amount received by the agent from principal k for taking action a is $(1 - \mu)\pi(z^k = 0|a)\lambda\phi^k - \mu\pi(z^k = 1|a)\lambda\phi^k$. Hence the overall expected utility of the agent for the action $k \neq 0$ is $(1 - \pi)(1 - \mu)\lambda\phi^k - \pi\mu\lambda\phi^k + (1 - \pi_b)(1 - \mu)\lambda\phi^{-k} - \pi_b\mu\lambda\phi^{-k} - c$ and for action 0 it is $[(1 - \pi_b)(1 - \mu) - \pi_b\mu] \lambda(\phi^k + \phi^{-k})$.

3.1. Agent Incentives

Because the effect of effort by the two principals is symmetric, between actions 1 and 2 the agent prefers the action of the principal k who provides the greater effort. Suppose the efforts are $\phi^k \geq \phi^{-k}$. When does the agent prefer choosing k to inaction? The utility advantage of action k over the action 0 is

$$\begin{aligned}(1 - \pi)(1 - \mu)\lambda\phi^k - \pi\mu\lambda\phi^k + (1 - \pi_b)(1 - \mu)\lambda\phi^{-k} - \pi_b\mu\lambda\phi^{-k} - c& \\ - [(1 - \pi_b)(1 - \mu) - \pi_b\mu] \lambda(\phi^k + \phi^{-k})&\end{aligned}$$

[7]Conceptually there is no reason a principal should not be able to reward a bad signal and punish a good signal. However in the first stage game, that is, given agent optimal play in the second stage, not rewarding the bad signal or not punishing the good signal strictly reduces costs and regardless of the strategy of the other principal cannot increase the probability the agent takes a less favorable action. Hence strategies of rewarding on bad or punishing on good signals are strictly dominated in the first stage game and so are irrelevant.

which may be written as

$$(1-\pi)(1-\mu)\lambda\phi^k - \pi\mu\lambda\phi^k - c - \left[(1-\pi_b)(1-\mu) - \pi_b\mu\right]\lambda\phi^k.$$

This does not depend on ϕ^{-k}, because the amount of punishment or reward from principal $-k$ is the same whether the agent chooses action k or 0. From the expression above it follows that the agent strictly prefers inaction if and only if the efforts of both principals fall below a threshold

$$\phi^k < c/\left[\lambda(\pi_b - \pi)\right] \equiv \varphi,\ \ k = 1, 2.$$

Notice that the agent prefers inaction if the principals' effort levels are low *independently of* μ. This is why the equilibria in the theorem of the next section share fundamental features in the rewards and punishments cases. We comment on the differences after stating the result.

3.2. Principals' Willingness to Pay

Observe that principal k's marginal cost of providing effort when the agent chooses the preferred option k is $\gamma \equiv (1-\pi)(1-\mu) + \pi\mu$. The most effort principal k is willing and able to choose to get the preferred outcome k over any other outcome is $W^k = \min\{\Phi^k, \nu^k/\gamma\}$. We refer to this as principal k's *willingness*. Notice that $\Phi^1 > \Phi^2$ and $\nu^1 > \nu^2$ imply $W^1 > W^2$: the advantaged principal has a strictly higher willingness.

3.3. The Unique Equilibrium

The canonical example delivers precise predictions under all parameter configurations. We summarize these in the following theorem.

Theorem 2. *The canonical example has a unique equilibrium.*

a) If $W^1 < \varphi$*, the principals choose* 0 *effort and the agent chooses inaction.*

b) If $W^1 > \varphi > W^2$*, the disadvantaged principal chooses* 0 *effort, the advantaged principal chooses* φ *and the agent chooses* 1*.*

c) If $W^2 > \varphi$*, equilibrium is in mixed strategies; it has support* $\{0\} \cup [\varphi, W^2]$*, the disadvantaged principal gets* $v^2 = 0$ *and the advantaged principal gets* $v^1 = \nu^1 - \gamma W^2 = \nu^1 - \min\{\nu^2, \gamma\Phi^2\} > 0$*. In this equilibrium action* 0 *is not used. The cumulative distribution of effort level is*

$$F^k(\phi^k) = \frac{v^{-k} + \mu\phi^k}{\nu^{-k} + (\mu - \gamma)\phi^k}$$

for $\phi^k \in [\varphi, W^2)$*, with* $F^2(0) = F^2(\varphi), F^1(0) = 0$ *and* $F^k(W^2) = 1 - \lim_{\phi^k \to W^2} F^k(\phi^k)$*.*

Proof. If $W^1 < \varphi$, then both principals optimally select 0, irrespective of the other's choice. The agent's choice in equilibrium must therefore be inaction.

If $W^1 > \varphi > W^2$ principal 1 can guarantee $a = 1$ by choosing effort φ since W^2 is lower than that. Since the disadvantaged principal would never select positive effort in equilibrium in this case, the unique equilibrium must involve the advantaged principal selecting φ and the agent choosing 1.

Now consider the case of $W^2 > \varphi$. The key thing is that any bid below φ must be at 0, the reason being that for any $\phi^k < \varphi$ principal k gets zero (the agent will choose either$-k$ or 0) and if $\phi^k > 0$ the bid costs $[(1-\pi_b)(1-\mu) + \pi_b\mu]\phi^k > 0$.

The remainder of the argument is a standard all-pay auction argument. Nobody bids more than the "top" $W^2 = \min\{\Phi^2, \nu^2/\gamma\}$. There cannot be a positive probability of a tie below the top: one of the two should raise the atom slightly. There cannot be a gap (zero probability interval) at or above φ since then it would pay one to shift bids from the top of the gap to the bottom. Since bidding then has to go all the way down to φ one has to get zero . It must be the disadvantaged principal since the advantaged principal can guarantee a positive return by bidding just above the top. The disadvantaged principal getting zero forces bidding to the top, since otherwise there is a positive return by bidding a bit above the maximum bid.

We can then work out the equilibrium strategies. The indifference condition for k and bids at and above φ is given by $F^{-k}(\phi^k)(\nu^k - \gamma\phi^k) - (1 - F^{-k}(\phi^k))\mu\phi^k = v^k$ so that

$$F^{-k}(\phi^k) = \frac{v^k + \mu\phi^k}{\nu^k + (\mu - \gamma)\phi^k}.$$

The disadvantaged principal must get 0 from bidding 0 so must lose for sure, meaning that the advantaged principal never bids 0. From this it easily follows that

$$F^2(0) = F^2(\varphi) = \frac{\lambda(\pi_b - \pi)v^1 + \mu c}{\lambda(\pi_b - \pi)\nu^1 + (\mu - \gamma)c}.$$

Note that there is an atom at φ for the advantaged principal of

$$F^1(\varphi) = \frac{\mu c}{\lambda(\pi_b - \pi)\nu^2 + (\mu - \gamma)c}$$

when $\mu = 1$. When $\mu = 0$, the advantaged principal has an atom at W^2 with probability $1 - \lim_{\phi^1 \to W^2} F^1(\phi^1)$

Furthermore, inaction happens with 0 probability. □

Notice that the uniqueness result we obtain here is in sharp contrast to the unwieldy multiplicity typical of common agency games. So, our subsequent results do not depend on the validity of unmodeled equilibrium selection arguments. Observe also that when both principals choose to be active - case (c) - they both randomize, in contrast to the pure strategy equilibria that are typically the focus in common agency games. This has observable implications. The agent in our equilibrium in (c) does indeed select the action of the "losing" principal with positive probability. Nevertheless, similar to the finding in the common agency literature, the "losing" principal ends up with 0 expected payoff, while the "winning" principal gets the difference between the surplus generated by the two ideal actions of the principals.

It may seem at this point that expected payoffs stated in part (c) do not depend on whether efforts translate to rewards or punishment. This is only partly correct. Whether its a reward or a punishment environment determines the value of γ, which in turn potentially affects willingness. So if reward and punishment both lead to willingness values that fall in part (c) of the theorem, then indeed the expected payoffs remain the same. But it could be that with punishment we are in part (c) and both principals are active while with rewards we are in part (a) and both principals effectively drop out. We discuss this further in section 3.6.

Observe that unlike in case (b) where the agent never selects the disadvantaged principal 2, in case (c) the agent does indeed select 2 with positive probability. Nevertheless, the disadvantaged principal ends up with 0 expected payoff. In cases (b) and (c) rewards and punishments both occur with positive probability in the corresponding equilibria.

3.4. Comparative Statics

We can now discuss how the equilibrium effort choices of the two principals relate to each other and change in response to changes in the underlying parameters. For the following result we focus on the case when both principals are active, $W^2 > \varphi$.

Theorem 3. *Assume $W^2 > \varphi$. a) In equilibrium, the mixed strategy of effort used by the advantaged principal weakly first order stochastically dominates that used by the disadvantaged principal, $F^2 \geq F^1$.*

b) The equilibrium mixed strategy of effort used by the advantaged principal for a higher ν^2 weakly first order stochastically dominates that for a lower ν^2.

c) The equilibrium mixed strategy of effort used by the disadvantaged principal for a higher W^2 weakly first order stochastically dominates that for a lower W^2.

Proof. Stochastic dominance of advantaged $F^2 \geq F^1$

$$\frac{\nu^1 - \min\{\nu^2, \gamma\Phi^2\} + \mu\phi^k}{\nu^1 + (\mu-\gamma)\phi^k} \geq \frac{\mu\phi^k}{\nu^2 + (\mu-\gamma)\phi^k}$$

$$\left(\nu^1 - \min\{\nu^2, \gamma\Phi^2\} + \mu\phi^k\right)\left(\nu^2 + (\mu-\gamma)\phi^k\right) \geq \mu\phi^k\left(\nu^1 + (\mu-\gamma)\phi^k\right)$$

$$\left(\nu^1 - \min\{\nu^2, \gamma\Phi^2\}\right)\left(\nu^2 + (\mu-\gamma)\phi^k\right) + \mu\phi^k\left(\nu^2 + (\mu-\gamma)\phi^k\right) \geq \mu\phi^k\left(\nu^1 + (\mu-\gamma)\phi^k\right)$$

$$\left(\nu^1 - \min\{\nu^2, \gamma\Phi^2\}\right)\left(\nu^2 + (\mu-\gamma)\phi^k\right) \geq \mu\phi^k\left(\nu^1 - \nu^2\right)$$

true if

$$\left(\nu^2 + (\mu-\gamma)\phi^k\right) \geq \mu\phi^k$$

$$\left(\nu^2 - \gamma\phi^k\right) \geq 0$$

which is true in the domain $\phi^k \leq W^2 = \min\{\Phi^2, \nu^2/\gamma\}]$ □

The theorem describes the intuitive result that making the principals more similar, by increasing ν^2 or W^2 results in more aggressive competition between them. Observe that this is true irrespective of whether effort translates to punishments as opposed to rewards (the parameter μ).

3.5. Joining results: Two masters or one?

In a variety of settings agents have the ability to choose which groups to join. We can use the results from section 3.3 to endogenize this choice. To do so, we extend the basic game of section 3 by supposing that prior to playing that game the agent chooses one of the two principals or both or neither, to join. Joining with principal k provides an additive benefit of $b^k > 0$ where $b^1 \geq b^2$. Following this decision the basic game is played between the agent and the subset of principals with whom the agent has joined.[8] A subgame perfect equilibrium of this supergame is an equilibrium of the basic game corresponding to each subset of principals together with a choice of a subset of principals by the agent with the property that the utility from the supergame plus the benefit of joining the principals is maximized. Since each basic game has an equilibrium, the existence of a subgame perfect equilibrium of the supergame game follows immediately.

Theorem 4. *With rewards both principals are joined. With punishment:*

(a) if $W^1 < \varphi$ then both groups are joined;

(b) If $W^1 > \varphi > W^2$ then: if $b^1 > c\pi_b/(\pi_b - \pi)$ both groups are joined; if $b^1 < c\pi_b/(\pi_b - \pi)$ then only the disadvantaged principal is joined.

(c) If $W^2 > \varphi$, then if $\lambda\Phi^2 \leq b^2$ both principals are joined.

(d) If $W^2 > \varphi$, for fixed values of the other parameters there exists $\overline{\lambda}$ such that for $\lambda > \overline{\lambda}$ only the advantaged principal is joined if $b^1 > c\pi_b/(\pi_b - \pi)$ and none of the principals is joined otherwise.

[8]Strictly speaking the basic game is defined only for the case of two principals. If the agent joins one principal the obvious resulting game has the same cost, utility and signal functions as the two-principal game.

Proof. First consider equilibrium where only principal k is joined. If $W^k < \varphi$ that principal will not bid so the agent gets b^k. If $W^k > \varphi$ then he is paid just enough to be indifferent between 0 and k, that is the principal provides effort $c/\left[\lambda(\pi_b - \pi)\right]$ and if the agent chooses inaction in the reward case receives $b^k + (1-\pi_b)c/(\pi_b - \pi)$ and in the punishment case $b^k - \pi_b c/(\pi_b - \pi)$

With rewards, note first that the agent gets 0 from not joining any group. Now consider the case of $W^1 < \varphi$. He gets b^k from joining group k alone and by Theorem 2, gets $b^1 + b^2$ from joining both. Therefore he joins both. With $W^1 > \varphi > W^2$ the agent gets $b^1 + c(1-\pi_b)/(\pi_b - \pi)$ from joining group 1 alone, b^2 from joining group 2 alone and $b^1 + b^2 + c(1-\pi_b)/(\pi_b - \pi)$ from joining both. Again, the agent optimally joins both groups. Finally consider the case $W^2 > \varphi$. The agent gets $b^1 + c(1-\pi_b)/(\pi_b - \pi)$ from joining group 1 alone, and $b^2 + c(1-\pi_b)/(\pi_b - \pi)$ if he joins group 2 alone. If the agent joins both groups then we are in the mixed equilibrium case. Observe that the advantaged principal always chooses rewards no less than φ. The only choice of the disadvantaged principal that is less than φ is when he selects 0 and in this case the agent chooses action 1. In other words, no matter the realization of rewards from the mixed strategies used by the principals, the agent gets an expected reward of at least $(1-\pi_b)\lambda c/\left(\lambda(\pi_b - \pi)\right)$. So by joining both groups the agent gets no less than $b^1 + b^2 + c(1-\pi_b)/(\pi_b - \pi)$. As a result, the agent joins both groups.

Turning to the punishment case we first observe that if the agent joins only principal k he would never choose $a = -k$ because the alternative $a = 0$ has lower cost and equal probability of punishment. So if $W^k < \varphi$ then $\phi^k = 0$, $a = 0$ and the agent's payoff is b^k; if $W^k \geq \varphi$ then $\phi^k = \varphi$, $a = k$ and the agent's payoff is $b^k - \pi\lambda\varphi - c = b^k - c\pi_b/(\pi_b - \pi)$.

(a) If $W^1 < \varphi$ then by joining group k only there is no punishment and the agent chooses 0 and gets b^k ; if he joins both groups by Theorem 2(a) the situation is the same, no punishment and $a = 0$ - thus he gets $b^1 + b^2 > b^k$.

(b) If $W^1 > \varphi > W^2$ then if he joins group 2 alone, he gets no punishment and payoff b^2; if he joins only 1 he gets $b^1 - c\pi_b/(\pi_b - \pi)$; if he joins both groups Theorem 2(b) says he takes $a = 1$ and gets punishment $\pi\lambda\varphi$ so his payoff is $b^1 + b^2 - c\pi_b/(\pi_b - \pi)$. So joining both groups is always better than joining only group 1. Therefore, if $b^1 < c\pi_b/(\pi_b - \pi)$ then only group 2 is joined and otherwise both groups are joined.

(c) Suppose now $W^2 > \varphi$ and $\lambda\Phi^2 \leq b^2$. If the agent joins one principal then the agent receives at most $b^1 - c$. If the agent joins both principals let ξ be the equilibrium expected cost of punishment. So the agent gets $b^1 + b^2 - c - \xi$ and the net benefit of joining both principals is at least $b^2 - \xi$. To establish the result we must show this is strictly positive. As we are by assumption in the mixed case the utility of the advantaged principal is $v^1 = \nu^1 - \gamma W^2 = \nu^1 - \min\{\nu^2, \gamma\Phi^2\} \geq \nu^1 - \gamma\Phi^2$. As we have assumed $\lambda\Phi^2 \leq b^2$ it follows that $v^1 \geq \nu^1 - \gamma b^2/\lambda$. Let ω denote the equilibrium probability that the agent chooses the advantaged principal's preferred action. Then since the disadvantaged principal gets 0 we also know that the utility of the advantaged principal is equal to the total benefit received by both principals - equal to the expected value of the prize minus the expected cost of the punishment: $v^1 = \omega\nu^1 + (1-\omega)\nu^2 - \xi/\lambda$. Since $v^1 \geq \nu^1 - \gamma b^2/\lambda$ it follows that $\omega\nu^1 + (1-\omega)\nu^2 - \xi/\lambda \geq \nu^1 - \pi b^2/\lambda$ and we may rearrange this as $\pi b^2 - \lambda(1-\omega)(\nu^1 - \nu^2) \geq \xi$. Since $b^2 > \pi b^2 - \lambda(1-\omega)(\nu^1 - \nu^2)$ it follows that $b^2 > \xi$, which as we showed is the condition for the agent to strictly prefer joining both principals.

(d) Again we are by assumption in the mixed case. Hence there is a unique equilibrium and for $\phi^k \in [\varphi, W^2]$, the disadvantaged principal gets $v^2 = 0$ and the advantaged principal gets $v^1 = \nu^1 - \gamma W^2$, and the cumulative distribution of effort level is

$$F^k(\phi^k) = \frac{v^{-k} + \mu\phi^k}{\nu^{-k} + (\mu - \gamma)\phi^k}.$$

The only parameter here that depends upon λ is the lower bound $\varphi = c/[\lambda(\pi_b - \pi)]$ and we see that as $\lambda \to \infty$ we have $\varphi \to 0$. Hence for sufficiently large λ we have $\varphi < W^2/2$. So, for such large λ there is a positive number Q such the probability that principal k chooses an effort level at least equal to $W^2/2$ is greater than or equal to Q. Hence the expected punishment cost to the agent is at least $\lambda\pi Q^2 W^2/2$ which for sufficiently large λ leads to arbitrarily negative payoffs. By contrast if a single principal k is joined the agent gets $b^k - c\pi_b/(\pi_b - \pi)$, for all such high values of λ. Further since $b^1 \geq b^2$ the agent would prefer principal 1. So for such values of λ the agent would only join the advantaged principal if $b^1 > c\pi_b/(\pi_b - \pi)$ and neither of the principals otherwise. □

Our results show that the question of which groups to join in the standard rewards-only common agency environment is trivial. The agent always joins both groups. With punishment, group membership depends more subtly on the punishment and signal technology.

Consider first the intermediate case $W^1 > \varphi > W^2$ and $b^1 < c\pi_b/(\pi_b - \pi)$. In this case the agent does not want to join the advantaged principal because the principal will try to punish him to induce the action 1 and this will cost more than the benefit of joining. Joining both principals is the same - but the disadvantaged principal finds it not worthwhile to try to induce the action 2, so joining just that principal brings a benefit. Notice that the lack of commitment on the part of the principals plays a key role here: the agent joins the principal who brings less benefit because that principal will leave him alone while the advantaged principal would impose punishments inducing him to engage in costly effort.[9]

[9] Notice also that while we have set $v^k(0) = 0$ this is just a normalization: zero effort by the agent could well provide a benefit to the principals so that, for example, it could be that the benefit $v^1(0) = v^1(2) >$

Excluding this case, suppose that punishments by the disadvantaged principal is limited to exclusion from the benefits of joining a principal then both groups are joined. In this case it must be that $\lambda\Phi^2$ the greatest punishment that can be issued by the disadvantaged principal is no greater than b^2 the benefit of joining. Hence in all cases both principals are joined. This is relevant to social groups in which punishment takes place through partial exclusion from group activities and there is no fee for joining. So, it should not surprise us if we find two social groups with competing demands on a member's time to nevertheless have a considerable overlap in membership.

By contrast, if severe punishments are possible at low cost then the competition between the principals is damaging enough and it does not pay to join both. Here again inability to commit on the part of the principals plays a key role. Although the agent is quite willing to accept the cost of effort $c(1)$ in exchange for the benefit b^k the cost of the competitive punishment by the two principals swamps any possible benefit of joining both groups. Groups that can use violence cheaply to punish erring members would fit this category. Violent gangs membership, by this logic, should exhibit little if any overlap.

3.6. Welfare

Needless to say, the agent prefers that the principals use rewards rather than punishments. As the disadvantaged principal gets an expected utility of 0 regardless, he is clearly neutral. What about the advantaged principal?

Theorem 5. *Assume $\pi < 1/2$, and suppose $\Phi^1 > \varphi$. Then the advantaged principal never strictly prefers reward to punishment. If π is sufficiently small then the advantaged principal strictly prefers punishment to reward.*

Proof. Recall that $\mu = 1$ corresponds to punishment and that $\gamma \equiv (1-\pi)(1-\mu)+\pi\mu$. We use subscript r for the reward case and the subscript p for the punishment case - so for example $W_p^k = \min\{\Phi^k, \nu^k/\gamma_p\}$, where $\gamma_p = \pi$. Since $\pi < 1/2$ it follows that $\gamma_p < \gamma_r$. Suppose first that $W_p^1 < \varphi$. Since $W_p^1 \geq W_r^1$ we have $W_r^1 < \varphi$ and so the advantaged principal gets 0 regardless of whether punishments or rewards are used. Suppose next that $W_p^1 > \varphi > W_p^2$. In this case with punishment the advantaged principal gets $\nu^1 - \pi\varphi$ while with rewards the advantaged principal either gets either $\nu^1 - (1-\pi)\varphi$ or 0. So, the advantaged principal does strictly better with punishment in this case. Finally suppose $W_p^1, W_p^2 > \varphi$. Then the advantaged principal gets $\nu^1 - \min\{\nu^2, \gamma_p\Phi^2\}$ in the case of punishments. In the case of rewards his payoff could be one of three. If $W_r^1 < \varphi$ then it is 0, which is less than $\nu^1 - \nu^2$

$v^2(0) = v^2(1) > 0$ and $\nu^k = v^k(k) - v^k(0)$ in which case joining the advantaged principal and providing no effort would be strictly better for both than joining the disadvantaged principal. Never-the-less the inability to commitment on the part of the advantaged principal leads the agent to join with the lower value principal.

since the latter is strictly positive by assumption. If $W_r^1 > \varphi > W_r^2$ then his payoff is $\nu^1 - (1-\pi)\varphi$. Since $\varphi > W_r^2$ we have $\nu^2 < (1-\pi)\varphi$,and so again the principal prefers punishments. Lastly if $W_r^2 > \varphi$ then his payoff is $\nu^1 - \min\{\nu^2, \gamma_r \Phi^2\}$ which must be no greater than $\nu^1 - \min\{\nu^2, \gamma_p \Phi^2\}$ since $\gamma_p < \gamma_r$. So, the advantaged principal never strictly prefers rewards to punishment.

Now consider that as $\pi \to 0$ we have $\gamma_p \to 0$ and $\gamma_r \to 1$. Hence $W_p^1 \to \Phi^1 > \varphi$. Hence, we are either in the case $W_p^1 > \varphi > W_p^2$ where we already showed that the advantaged principal does strictly better with punishments; or we are in the case $W_p^2 > \varphi$ in which case as $\pi \to 0$ the advantaged principal gets ν^1 with punishments and $\nu^1 - \min\{\nu^2, \Phi^2\}$ with rewards, so punishment delivers a strictly greater payoff again. □

The intuition behind this result is that if you are going to win the contest it is better to pay when you lose (punishments) rather than when you win (rewards). Nevertheless, notice that rewards and punishments are both costly - it is only *in equilibrium* that rewarding becomes costlier. The result is interesting, especially because in real-world social aggregations marginalization weighs more heavily than costly rewarding.

4. Compromise

In the canonical example inaction is preferred by the agent when $c > 0$ and this forces the principals to bid actively for the favor of the agent. But for either principal k, inaction is equivalent to the opponent's favorite action - both in terms of utility and observation (signal probabilities). We now wish to model the action that neither principal favors the most (action 0) as a compromise, again in terms of payoffs and observation. We then assume that action $a = 0$ yields each principal a payoff $0 \leq \eta^k < \nu^k$ that is in-between those generated by the actions 1 and 2. Observationally, we assume that compromise generates the bad signal for principal k with a higher probability than k, but lower than $-k$: if the agent takes the compromise action 0 the probability of getting the bad signal 1 is π_c with $\pi < \pi_c < \pi_b < 1$.

A couple of remarks are due. First, the payoff structure in this section is different in an important way to that in the canonical section. An example of the latter involves principals competing for the agent's time where if the agent chooses to devote his time elsewhere a principal is unaffected by exactly where that time is spent. In this section we allow for more externalities, in that as a principal, if an agent works for the other principal it hurts you more than if he were to remain neutral (choose compromise). Consider, for instance, a setting of civil conflict in which each faction, conditional on an agent not joining, prefers he remains unaffiliated to joining the opposite faction.

Second, the information structure in the canonical example is such that a principal cannot tell if the agent chose 0 or sided with the other principal. Hence, the principals have no ability to induce the choice of 0 instead of the action favoured by the other principal. The information structure of this section, by contrast, allows principals to give incentives for compromise. We are interested in whether this translates to the emergence of compromise in equilibrium.

To keep things simple we focus on the case in which $c = 0$ so that the agent is indifferent across all actions. We also assume that $\Phi^k > \nu^k/\gamma$ so that the effort constraint does not bind in equilibrium. As a benchmark recall from Theorem 2 what is the equilibrium in the canonical model in this case: it is unique, it is in mixed strategies, the support for both principals is $[0, \nu^2/\gamma]$ from zero to the most effort principal 2 is willing and able to provide to get the agent to choose $a = 2$ rather than $a = 1$, the disadvantaged principal gets zero and the advantaged principal gets $\nu^1 - \nu^2$ so that the social surplus is also $\nu^1 - \nu^2$. With our new assumptions about the compromise action, we get the following results:

Theorem 6. *(a) If $\pi_c > (\pi + \pi_b)/2$ then there is a unique equilibrium. In this equilibrium compromise is not used and the strategies and payoffs to the principals and agent are identical to those in the canonical model.*

(b) If $\pi_c < (\pi + \pi_b)/2$ compromise occurs with positive probability in every equilibrium. The equilibrium distributions have at most one atom for one principal at zero.

(c) If compromise is efficient in the sense that $\nu^1 < \eta^1 + \eta^2$, then as $\pi_c \to \pi$ the probability that the compromise action occurs converges to one uniformly over equilibria.

Proof. The utility advantage of action k over the action 0 is

$$(1-\pi)(1-\mu)\lambda\phi^k - \pi\mu\lambda\phi^k + (1-\pi_b)(1-\mu)\lambda\phi^{-k} - \pi_b\mu\lambda\phi^{-k} - [(1-\pi_c)(1-\mu) - \pi_c\mu]\,\lambda(\phi^k + \phi^{-k})$$

which simplifies to $\lambda[(\pi_c - \pi)\phi^k - (\pi_b - \pi_c)\phi^{-k}]$; it follows that the agent would strictly prefer $a = 0$ to action k if and only if

$$\phi^k < \frac{\pi_b - \pi_c}{\pi_c - \pi}\phi^{-k} \equiv \beta\phi^{-k}$$

Therefore he would strictly prefer $a = 0$ to both other actions if and only if

$$\beta > \frac{\phi^1}{\phi^2} > \frac{1}{\beta}.$$

The agent would strictly prefer both actions to $a = 0$ if and only if the condition above hold with the inequalities reversed.

Proof of part (a): If $\beta < 1/\beta$, which is equivalent to

$$\pi_c > \frac{\pi + \pi_b}{2}$$

the agent strictly prefers one of the two non-zero actions to $a = 0$ for any ϕ^1, ϕ^2 with at least one positive effort level. Furthermore, for all such ϕ^1, ϕ^2, the agent prefers the action k with the higher ϕ^k. Finally, with $\phi^1 = \phi^2 = 0$, the agent is indifferent across all three actions. This means that following any pair of pure strategies by the principals in the first period, the problem faced by the agent is identical to that in the canonical model with $c = 0$. So, against any strategy of the opposing principal and any second stage equilibrium choice of the agent, the payoff to a principal from a given effort level is exactly as in the canonical model unless both effort levels equal 0. Now, in equilibrium the probability of a tie at 0 must be 0, since one of the principals would be strictly better of raising his effort slightly higher. The same set of all-pay auction arguments that were used for the canonical example then apply directly to this case and deliver the same unique equilibrium.

Proof of part (b1) at most one atom at zero: Suppose that there is a positive atom at $\hat{\phi}^k > 0$. Consider that the opponent cannot have a mass point at either

$$\underline{\phi}^{-k} = \frac{1}{\beta}\hat{\phi}^k$$

where the agent is indifferent between choosing k and compromise nor at

$$\overline{\phi}^{-k} = \beta\hat{\phi}^k$$

where the agent is indifferent between choosing $-k$ and compromise: if there was an atom at one of those bids one of the two principals would wish to break the tie by increasing the atom slightly. Next suppose that $-k$'s equilibrium distribution has two gaps directly below each of $\underline{\phi}^{-k}, \overline{\phi}^{-k}$. In this case k should lower the atom slightly. Suppose then that $\tilde{\phi}^{-k} \in \{\underline{\phi}^{-k}, \overline{\phi}^{-k}\}$ has no gap directly below it. Then if $-k$ takes a sufficiently small neighborhood directly below $\tilde{\phi}^{-k}$ and moves it to a mass point above $\tilde{\phi}^{-k}$ but sufficiently close then at trivial cost $-k$ will have either converted a loss to a compromise or a compromise to a victory. This increases utility so is impossible. We conclude that there was not a positive atom in the first place. It also cannot be that both principals have an atom at zero since then one of them should increase the bid slightly to insure victory against the other principal's atom.

Proof of part (b2) no compromise: Suppose there is no compromise on the equilibrium path. Take a positive bid $\hat{\phi}^1$ for principal 1 such that every neighborhood has a positive probability. Then any bid by 2 in the interval

$$\beta\hat{\phi}^1 > \phi^2 > \frac{1}{\beta}\hat{\phi}^1$$

results in compromise, hence 2 does not bid in this interval since there is no compromise on the equilibrium path. Since there is no compromise on the equilibrium path any equilibrium value of ϕ^1 in the same interval must lose to any equilibrium bid by 2 greater than ϕ^1 and win against any equilibrium bid by 2 less than ϕ^1. Since the set of higher and lower equilibrium bids by are the same for any ϕ^1 in this interval (as 2 does not play there in equilibrium) it follows that the probability of winning or losing for equilibrium bids in the interval is the same for all ϕ^1. Hence only $\hat{\phi}^1$can be an equilibrium bid: the entire support in the close interval must be an atom at this point. But there are no positive atoms by part (b1) so principal 1 must play zero with probability one The same reasoning applies to principal 2. Since both principals cannot have an atom so we conclude that any equilibrium must involve compromise with positive probability.

Proof of part (c): In equilibrium no principal will bid more than ν^1/γ. This implies that against any equilibrium strategy of an opponent if k bids

$$\phi^k > \frac{1}{\beta}\frac{\nu^1}{\gamma}$$

then at worst a compromise will occur. It follows that k's utility in equilibrium is at worst

$$\eta^k - ((1-\pi_c)(1-\mu) + \pi_c\mu)\,\frac{1}{\beta}\frac{\nu_1}{\gamma}.$$

As $\pi_c \to \pi$ this converges to η^k because $\beta \to \infty$. Consequently equilibrium social surplus converges to $\eta^1 + \eta^2$ which by assumption is greater than v^1. Hence if the probability of compromise is bounded away from 1 then equilibrium social surplus could not converge to $\eta^1 + \eta^2$. □

The key implication of our results is that it is the observational features of compromise, as opposed to the payoffs it generates, that has a first order effect on equilibrium outcome. If compromise is observationally closer to the least favoured action then it has no impact on equilibrium whatsoever, no matter how high the social gains compromise brings. On the other hand, if compromise is observationally closer to the favoured action, then compromise must occur with positive probability. In this case, if compromise is the efficient action, this probability converges to one at the limit. Observe also that the equilibrium implications of compromise we derive are independent of whether effort translates into rewards or punishment.

5. Robust Inefficiency in Common Agency Games

In this section we confine attention to the case of rewards only (in our previous notation $\mu = 0$) to relate our analysis to the existing literature on common agency games. In the case of rewards the information structure of the earlier sections is incompatible with standard common agency games because we allow for only two signals although there are three agent choices. Hence we now extend the rewards-only model to the case of three signals. This makes the standard common agency game a limit as the noise in the signals vanishes - or put differently we can regard the model of the current section as a perturbation of the standard common agency game.

In the perturbed setting, in which a principal may mistakenly observe any possible outcome, rewarding any signal now has a direct effect on the principal's payoff. Indeed, most equilibria described in Bernheim and Whinston (1986a) as using "non-serious" strategies are ruled out in the presence of imperfect monitoring, hence the error free limit of our perturbation makes a sharp refinement of the set of common agency equilibria. However, it turns out that this refinement does not rule out certain inefficient outcomes. Therefore, such inefficiency is not merely the result of miscoordination from not requiring principals to "put their money where their mouth is." On the contrary, such inefficiency seem characteristic of common agency games.

Bernheim and Whinston (1986a) do not offer a precise definition of "serious" strategies. They simply describe bids on actions that do not get selected in equilibrium and that do not reflect the relative preference of the principal as not serious. The notion of truthful strategies is clearly formalized, however, and basically requires a principal's bid profile to reflect his relative preference across the different actions. By contrast, equilibrium analysis of our perturbation does not force principals' bids to reflect their relative preference. Instead, it ensures that all bids are relevant to a principal's payoff since no matter which action the agent takes the principal will be forced to pay up the bid with positive probability. In short, while Bernheim and Whinston (1986a) formalized "serious" strategies by requiring a principal's bid schedule to reflect his relative preference, we do so by ensuring that no bid is off equilibrium.

We find that when compromise is inefficient there is a unique equilibrium that in the limit is efficient and is outcome equivalent to both the truthful equilibrium of Bernheim and Whinston (1986a) and the natural equilibrium of Kirchsteiger and Prat (2001) in the common agency game. However when compromise is efficient there are at least two equilibria: one that in the limit is efficient and equivalent to the truthful equilibrium, and one that in the limit is inefficient and is equivalent to the natural equilibrium. Hence from the perspective of the noisy information structure considered in this paper when the truthful and natural equilibria differ there seems to be no good reason to rule out either one.

Since we wish to perturb complete information common agency games with three actions, we need to have a model with three signals as opposed to the two we have used so far. With two signals we assumed without loss of generality that rewards were given only for a good signal - this enabled us to study a single level of effort. Here we allow for the full three dimensions of effort possible with three signals. It turns out that in equilibrium the principals never invest effort towards their least favored action, thereby effectively making the principal's problem two dimensional.

5.1. Rewards with Three Signals

Specifically, we continue to study two principals and an agent who can take three actions $a \in \{0, 1, 2\}$. The payoff structure remains unchanged and we assume one-to-one transfers, that is $\lambda = 1$. So action $k \neq 0$ is worth $\nu^k > 0$ to principal k and action $-k$ is worth 0 and the action 0 is worth η^k with $\nu^k \geq \eta^k \geq 0$ so that principal k prefers the action k. We continue to assume that $\nu^1 > \nu^2$, and continue to call principal 1 the *advantaged principal.* In the appendix we show that the results here are robust to more general payoff structures. For the agent we continue to work in the indifference case $c(a) = 0$ for all actions.

There are now three signals $z \in A = \{0, 1, 2\}$. The signal has probability $1 - \pi$ of "being correct" in the sense that $\Pr(z = k|a = k) = 1 - \pi$ and an equal probability of taking a wrong value in the sense that for $j \neq k$ we have $\Pr(z = j|a = k) = \pi/2$. We assume that $0 < \pi < 1/2$.

Principal k commits to a reward schedule, with reward $R^k(j) \geq 0$ following signal j. Recall that we have assumed that a unit of reward costs a unit of effort. We also assume that the upper bounds on these effort levels are not binding.

Following the choice of a pure strategy by the two principals, $R^k(j)$ for $k \in \{1, 2\}$, a choice of $a \in \{0, 1, 2\}$ by the agent gives him an expected payoff of $(1-\pi)[R^1(a) + R^2(a)] + (\pi/2)\sum_{j\neq a}\sum_k R^k(j)$. If $a = k$, principal k's expected payoff is $\nu^k - (1 - \pi)R^k(k) - (\pi/2)[R^k(-k) + R^k(0)]$, if $a = -k$ it is $-(1 - \pi)R^k(-k) - (\pi/2)[R^k(k) + R^k(0)]$ and with $a = 0$ it is $\eta^k - (1 - \pi)R^k(0) - (\pi/2)[R^k(k) + R^k(-k)]$.

5.2. Two Types of Equilibria

We focus on two specific types of equilibria. The first one we label the *no compromise* strategy profile. The strategies involve principal k choosing to reward only upon observing signal k according to the distribution $F^k(r)$ where

$$F^1(r) = \frac{(\pi/2)r}{\nu^2 - (1 - (3\pi/2))r}, \quad F^2(r) = \frac{\nu^1 - \nu^2 + (\pi/2)r}{\nu^1 - (1 - (3\pi/2))r}$$

both with support $\left[0, \nu^2/(1 - \pi)\right]$. The agent simply picks the action that offers the higher reward and breaks ties in favor of action 1.

We will show that the no compromise profile is always an equilibrium. In the limit as $\pi \to 0$, principal 1 bids ν^2 for sure and so the agent selects $a = 1$ with certainty too. When compromise is inefficient in the sense that $\eta^1 + \eta^2 < \nu^k$ for both k, this is efficient and has the same outcome as the truthful equilibrium of Bernheim and Whinston (1986a). When compromise is efficient in the sense that $\eta^1 + \eta^2 > \nu^k$ for both k this is inefficient.

The second we label the $\epsilon-$*compromise* strategy profile. Both principals use pure strategies $R^k(j)$ that reward both k and the compromise action. The rewards are given by

$$R^1(1) = R^2(2) = \frac{\nu^1 - \eta^1 + \nu^2 - \eta^2 + \epsilon}{1 - \pi}$$

$$R^1(0) = \frac{\nu^2 - \eta^2 + \frac{\epsilon}{2}}{1 - \pi} \qquad R^2(0) = \frac{\nu^1 - \eta^1 + \frac{\epsilon}{2}}{1 - \pi}$$

The agent picks the action that brings the highest reward if unique. In case of a two-way tie between action 0 and $k \in \{1, 2\}$, the agent picks k. In case of a three-way tie the agent picks 0. In this strategy profile, the principals reward their most preferred action as well as the compromise action. There is no randomization by the principals, and the compromise action is selected with certainty.

5.3. Main Result

Our next result implies that when compromise is inefficient only the no-compromise profile is an equilibrium in the limit, and in this range it is efficient; but when compromise is efficient then both the $\epsilon-$compromise strategy and the no compromise profile are equilibria, and the latter is inefficient in that range. In other words, equilibrium analysis cannot rule out the aggressive no-compromise profile even in the presence of an attractive, efficient compromise option.

Theorem 7. *(a) The no compromise strategy profile is always an equilibrium.*

For small enough π

(b) if $\nu^1 < \eta^1 + \eta^2$ *and* ϵ *and* π/ϵ *are sufficiently small the* $\epsilon-$*compromise strategy profile is an equilibrium*

(c) if $\nu^2 > \eta^1 + \eta^2$ *the no compromise equilibrium is the only equilibrium.*

The proof of this result is in the Appendix.

5.4. Examples

To illustrate the content of Theorem 7 we discuss a slightly modified couple of examples used in Kirchsteiger and Prat (2001). The number in brackets refers to the action chosen by the agent in the relevant equilibrium.

Game 1; efficient action is 1				
		1	0	2
Payoffs	Principal 1	17	6	0
	Principal 2	0	5	12
Truthful equilibrium (1)	R^1	12	1	0
	R^2	0	5	12
Natural equilibrium (1)	R^1	12	0	0
	R^2	0	0	12
(Limit of) *no-compromise equilibrium (1)*	R^1	12	0	0
	R^2	0	0	$F^2(r) = \frac{5}{17-r}$, $r \in [0, 12]$
(Limit of) $\epsilon-$*compromise equilibrium*	None			

Game 2; efficient action is 0				
		1	0	2
Payoffs	Principal 1	17	11	0
	Principal 2	0	7	12
Truthful equilibrium (0)	R^1	11	5	0
	R^2	0	6	11
Natural equilibrium (1)	R^1	12	0	0
	R^2	0	0	12
(Limit of) *no-compromise equilibrium (1)*	R^1	12	0	0
	R^2	0	0	$F^2(r) = \frac{5}{17-r}$, $r \in [0, 12]$
(Limit of) $\epsilon-$*compromise equilibrium (0)*	R^1	11	5	0
	R^2	0	6	11

The only difference between the two games is in the payoffs that compromise brings. In game 2, compromise maximizes the social surplus while in game 1 it does not. The truthful equilibrium tracks this change in the compromise payoffs since the principals' relative preference across actions changes across the games. Indeed, not only are the principals' strategies in the truthful equilibrium different across the two games, but so is the action chosen by the agent; he always picks the socially optimal action. However, notice that each principal's favourite action along with the payoff it brings remains unchanged across the two games. Since this is precisely what matters in a natural equilibrium, it too remains unchanged.[10]

[10] A natural equilibrium is an equilibrium of a common agency game in which each principal rewards at most 1 action. Such equilibria always exist in common agency games and typically it is not unique. Kirchsteiger and Prat (2001) identify a particular type of natural equilibrium that they call maximum-

The limit of the no-compromise equilibrium closely tracks the natural equilibrium across both games. The agent's choice and principal 1's strategy are identical. While principal 2's is not exactly the same, it does have the key feature that principal 2 chooses to reward only one action, which is also his favourite. In this sense we believe that our analysis offers strong support for the notion of natural equilibrium. By contrast, the limit of the $\epsilon-$compromise equilibrium is identical to the truthful equilibrium in game 2, while the former does not even exist in game 1. From Theorem 7, we know that in game 1, for small enough noise, the no-compromise profile is the unique equilibrium. This may look like an argument against the truthful equilibrium, since in game 1 the unique limit equilibrium strategy profile of the perturbed game does not resemble a truthful strategy profile at all. However, a closer look reveals that the two are outcome equivalent, in that the outcome and the payoffs to the principals and agent are identical. So while a limit equilibrium strategy profile of our perturbed game does not always track the truthful equilibrium profile, a limit equilibrium outcome does always match the truthful equilibrium outcome. In this sense, we believe our analysis makes a case for truthful equilibrium too.

6. Conclusion

We study a common agency model in which two principals with conflicting preferences attempt to influence an agent through costly effort. Our framework allows this effort to translate into either rewards or punishments, the former common in political lobbying, the latter being more relevant for competing social groups. Adopting the more realistic assumption of noisy observation of the agent's action delivers sharp equilibrium predictions, in contrast to the common agency literature. If both principals care enough about their favoured action, we find that they both randomize over effort in equilibrium. The outcome, as a result, is random too, but in expected terms it delivers payoffs equal to each principal's marginal contribution to the social surplus, as in the Vickrey Clarke Grove mechanism. Also, the more similar the principals the more aggressively they compete.

In the case the agent can choose which and how many groups (principals) to join, our results show that we should expect overlapping group memberships in the case of social groups, which discipline members through the threat of partial exclusion from group activities; while in groups which can cheaply punish members to an extent beyond withholding the benefits of membership, say through violence, we should expect no overlap. With rewards, such as in political lobbying, the agent would optimally join multiple groups. Relatedly, the

conflict equilibrium, which always exists and is typically unique when the principals' preferences are not aligned. In a maximum conflict equilibrium with two principals, each principal ends up rewarding the action that brings the highest payoff. All our mention of natural equilibrium refers to this maximum-conflict equilibrium.

agent always prefers rewards to punishment. The principals on the other hand, if and when they have a strict preference, prefer punishment. We find that whether the introduction of a compromise alternative leads to less competition depends first on the signal technology and then on how compelling compromise is.

We use our assumption of noisy observation to perturb the standard common agency (rewards-only) model. The limit equilibrium prediction offers support to both truthful equilibrium as well as the competing natural equilibrium. Since the latter is often inefficient, this may offer a new perspective on economic and political activities such as lobbying.

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Appendix: Common Agency

We consider a more general payoff structure than that in the text. We continue to assume that principal k prefers action k the most and there is no indifference. His values from actions $0, 1, 2$ are $v^k(0), v^k(1), v^k(2) \geq 0$. We assume that $v^1(1)+v^2(1) > v^1(2)+v^2(2)$. The case in the text is with $v^k(0) = \eta^k, v^k(k) = \nu^k \geq \eta^k, v^k(-k) = 0$. As in the text, in the *no compromise* strategy profile principal k rewards only the most preferred action k according to the distribution $F^k(r)$ where

$$F^1(r) = \frac{(\pi/2)r}{v^2(2) - v^2(1) - (1 - (3\pi/2))r} \quad , F^2(r) = \frac{v^1(1) - v^1(2) - \big(v^2(2) - v^2(1)\big) + (\pi/2)r}{v^1(1) - v^1(2) - (1 - (3\pi/2))r}$$

both with support $\left[0, \frac{v^2(2)-v^2(1)}{1-\pi}\right]$. The agent simply picks the action that offers the higher reward and breaks ties in favor of action 1.

In the $\epsilon-$*compromise* strategy profile both principals use pure strategies $R^k(j)$ that reward both k and the compromise action. The size of the rewards are given by

$$R^1(1) = R^2(2) = \frac{v^1(1) - v^1(0) + v^2(2) - v^2(0) + \epsilon}{1 - \pi}$$

$$R^1(0) = \frac{v^2(2) - v^2(0) + \frac{\epsilon}{2}}{1 - \pi} \qquad R^2(0) = \frac{v^1(1) - v^1(0) + \frac{\epsilon}{2}}{1 - \pi}.$$

The agent picks the action that brings the highest reward if unique. In case of a two-way tie between action 0 and $k \in \{1, 2\}$, the agent picks k. In case of a three-way tie the agent picks 0. Thus if the $\epsilon-$compromise strategy profile is played the agent chooses compromise.

Theorem 8. *(a) The no compromise strategy profile is always an equilibrium.*

(b) For small enough π and ϵ with $\epsilon/\pi > v^1(1)-v^1(0)+v^2(2)-v^2(0)$, the $\epsilon-$compromise strategy profile is an equilibrium if and only if $v^k(k) + v^{-k}(k) < v^k(0) + v^{-k}(0)$ for both $k = 1, 2$.

(c) For small enough π and either $v^k(k) + v^{-k}(k) > v^k(0) + v^{-k}(0)$ for both $k = 1, 2$ or $a = 0$ is the least preferred outcome for at least one principal there is a unique equilibrium - the no compromise equilibrium.

Proof. In what follows it will be useful to consider the following 4-element partition of the set of all pure strategies available to a principal. Let $S^k_p = \{R^k(j)|R^k(0) = 0, R^k(-k) = 0\}$ be pure strategies that reward only the preferred action, $S^k_o = \{R^k(j)|R^k(k) = 0, R^k(-k) = 0\}$ be ones that reward only the compromise action, $S^k_b = \{R^k(j)|R^k(k), R^k(0) > 0, R^k(-k) = 0\}$ be pure strategies that reward both and $S^k_w = \{R^k(j)|R^k(-k) > 0\}$ collect the remaining pure strategies, which assign positive reward the other principal's favorite action (of course such strategies could reward the other actions too).

(a) With the no compromise strategy profile principal 1's expected payoff is $v^1(1) + v^2(1) - v^2(2)$, while principal 2's expected payoff is $v^2(1)$. It suffices to show that no pure strategy deviation can do better.

S_p^k: These strategies that reward only pure actions are of two types: those with

$$R_p^k(k) \leq \frac{v^2(2) - v^2(1)}{1 - \pi}$$

are in the support of the equilibrium distribution hence by construction principal k is indifferent to the equilibrium payoff. For the remaining deviations with

$$R_p^k(k) > \frac{v^2(2) - v^2(1)}{1 - \pi}$$

similarly to the proof in section 3, it can be verified that these do not represent an improvement for principal k.

S_o^k; Again we may consider two types: those with

$$R_o^k(0) \leq \frac{v^2(2) - v^2(1)}{1 - \pi}$$

may be compared to the pure strategy $\widetilde{R}_p^k(j)$ from S_p^k with $\widetilde{R}_p^k(k) = R_o^k(0)$. Notice that $\widetilde{R}^k(j)$ belongs in the support of the equilibrium distribution. Given $-k$'s strategy, deviating to $R_o^k(j)$ from $\widetilde{R}_p^k(j)$, either leaves the agent's action unchanged (if the agent was selecting $-k$) or induces the agent to select 0 instead of k. This also equalizes the expected cost to the principal from using either strategy. Since principal k strictly prefers k to 0 and the expected cost of the two strategies is identical, $R_o^k(j)$ is not a profitable deviation. For the remaining deviations with

$$R_o^k(0) > \frac{v^2(2) - v^2(1)}{1 - \pi}$$

the expected payoff would be less than $v^k(0) - (v^2(2) - v^2(1))$. To see why, note first that with such an $R_o^k(j)$, the reward on action 0 is strictly higher than any reward offered by principal $-k$ in his mixed strategy. So the agent would pick action 0 with certainty, bringing principal k an expected payoff of $v^k(0) - (1-\pi)R_o^k(0) < v^k(0) - (v^2(2) - v^2(1))$. For $k = 1$ this is strictly less than $v^1(1) - (v^2(2) - v^2(1))$ while for $k = 2$ this is strictly less than $v^2(1)$.

S_b^k: Consider first a deviation in S_b^k with $R_b^k(k) < R_b^k(0)$. If $R_b^k(j)$ is profitable then so must $R_o^k(j) \in S_o^k$ with $R_o^k(0) = R_b^k(0) + \epsilon$ for a small enough ϵ. To see this, suppose that with $R_b^k(j)$ and principal $-k$'s strategy, the agent picks 0 with probability q and $-k$ with probability $1 - q$. Notice that since $R_b^k(k) < R_b^k(0)$ and principal $-k$ rewards only his own action, the agent would never pick k. Principal k's expected payoff is $q\left(v^k(0) - (1-\pi)R_b^k(0) - (\pi/2)\, R_b^k(k)\right) + (1-q)\left(v^k(-k) - (\pi/2)\left(R_b^k(k) + R_b^k(0)\right)\right)$. If he were to deviate instead to $R_o^k(j) \in S_o^k$ with $R_o^k(0) = R_b^k(0) + \epsilon$, his expected payoff would be $\widetilde{q}\left(v^k(0) - (1-\pi)\left(R_b^k(0)+\epsilon\right)\right) + (1-\widetilde{q})\left(v^k(-k) - (\pi/2)\left(R_b^k(0)+\epsilon\right)\right)$ with $\widetilde{q} \geq q$. Now, if $R_b^k(j)$ is profitable then $v^k(0) - (1-\pi)R_b^k(0)$ must be no less than $v^k(-k)$ because otherwise principal k by playing $R_b^k(j)$ would be getting strictly less than his expected payoff from the no compromise profile. So, given that $v^k(0) - (1-\pi)R_b^k(0)$ is no less than $v^k(-k)$, for small enough $\epsilon$the deviation $R_o^k(j)$ is more profitable than $R_b^k(j)$. But we already ruled out any deviation in S_o^k. So the principal k cannot do better with $R_b^k(j)$. Now consider a deviation in S_b^k with $R_b^k(k) > R_b^k(0)$. If $R_b^k(j)$ is profitable then so must $R_p^k(j) \in S_p^k$ with $R_p^k(k) = R_b^k(k) + \epsilon$ for a small enough ϵ. To see this, suppose that with $R_b^k(j)$ and principal $-k$'s strategy, the agent picks k with probability q and $-k$ with probability $1-q$. Notice that since $R_b^k(k) > R_b^k(0)$ and principal $-k$ rewards only his own action, the agent would never pick 0. Principal k's expected payoff is $q\left(v^k(k) - (1-\pi)R_b^k(k) - (\pi/2)\,R_b^k(0)\right) + (1-q)\left(v^k(-k) - (\pi/2)\left(R_b^k(k) + R_b^k(0)\right)\right)$. If he were to deviate instead to $R_p^k(j) \in S_p^k$ with $R_p^k(k) = R_b^k(k) + \epsilon$, his expected payoff would be $\widetilde{q}\left(v^k(k) - (1-\pi)\left(R_b^k(k)+\epsilon\right)\right) + (1-\widetilde{q})\left(v^k(-k) - (\pi/2)\left(R_b^k(k)+\epsilon\right)\right)$ with $\widetilde{q} \geq q$. Now, if $R_b^k(j)$ is profitable then $v^k(k) - (1-\pi)R_b^k(k)$ must be no less than $v^k(-k)$ because otherwise principal k by playing $R_b^k(j)$ would be getting strictly less than his expected payoff from the no compromise profile. So, given that $v^k(k) - (1-\pi)R_b^k(k)$ is no less than $v^k(-k)$, for small enough $\epsilon$the deviation $R_p^k(j)$ is more profitable than $R_b^k(j)$. But we already ruled out any deviation in S_p^k. So the principal k cannot do better with $R_b^k(j)$.

A deviation in S_b^k with $R_b^k(k) = R_b^k(0)$ is ruled out with a similar argument. Following such a deviation, suppose the agent chooses action k with probability q_k, 0 with probability q_0 and $-k$ with $1 - q_k - q_0$. Principal k's expected payoff is

$$q_k\left(v^k(k) - (1-\pi)R_b^k(k) - (\pi/2)\,R_b^k(0)\right) + q_0\left(v^k(0) - (1-\pi)R_b^k(0) - (\pi/2)\,R_b^k(k)\right) \\ + (1-q_k-q_0)\left(v^k(-k) - (\pi/2)\left(R_b^k(k) + R_b^k(0)\right)\right).$$

For this deviation to give an expected payoff greater than the no compromise profile it must be that $v^k(k) - (1-\pi)R_b^k(k)$ is strictly greater than $v^k(-k)$. If this deviation is indeed profitable then so must $R_p^k(j) \in S_p^k$ with $R_p^k(k) = R_b^k(k) + \epsilon$ for a small enough ϵ. The expected payoff from $R_p^k(j)$ would be $\widetilde{q}\left(v^k(k) - (1-\pi)\left(R_b^k(k)+\epsilon\right)\right) + (1-\widetilde{q})\left(v^k(-k) - (\pi/2)\left(R_b^k(k)+\epsilon\right)\right)$ with $\widetilde{q} \geq q_k + q_0$. For small enough ϵ the deviation $R_p^k(j)$ is more profitable than $R_b^k(j)$. But again, we already ruled out any deviation in S_p^k. So the principal k cannot do better with $R_b^k(j)$.

S_w^k: Finally consider a deviation, $R_w^k(j)$ Suppose, given $R_w^k(j)$ and principal $-k$'s strategy, the agent chooses action k with probability q_k, 0 with probability q_0 and $-k$ with

$1 - q_k - q_0$. Principal k's expected payoff is

$$q_k \left(v^k(k) - (1-\pi) R^k_w(k) - (\pi/2)\, R^k_w(0) - (\pi/2)\, R^k_w(-k) \right) \\ + q_0 \left(v^k(0) - (1-\pi) R^k_w(0) - (\pi/2)\, R^k_w(k) - (\pi/2)\, R^k_w(-k) \right) \\ + (1 - q_k - q_0) \left(v^k(-k) - (1-\pi) R^k_w(-k) - (\pi/2) \left(R^k_w(k) + R^k_w(0) \right) \right).$$

Notice that q_k and q_0 can both be positive only if $R^k_w(k) = R^k_w(0)$. Let $a \in \{k, 0\}$ represent the action that gets positive probability if both q_k and q_0 are not positive. If both are positive, then let $a \in \{k, 0\}$ represent the action that generates the higher expected payoff for the principal, the higher $v^k(a) - (1-\pi) R^k_w(a)$. For $R^k_w(j)$ to be profitable, it must be that $v^k(a) - (1-\pi) R^k_w(a) > v^k(-k)$, since otherwise the principal would be even better off offering no reward at all, which we know to be false. Now consider a different deviation $R^k(j)$ with $R^k(a) = \max_j R^k_w(j) + \epsilon$ and $R^k(j) = 0$ for $j \neq a$.

With $R^k(j)$ his expected payoff becomes

$$\widetilde{q} \left(v^k(a) - (1-\pi) \left(R^k_w(a) + \epsilon \right) \right) + (1 - \widetilde{q}) \left(v^k(-k) - (\pi/2) \left(R^k_p(k) + \epsilon \right) \right)$$

with $\widetilde{q} > q_k + q_0$. This is strictly higher than the expected payoff from $R^k_w(j)$. So if $R^k_w(j)$ is profitable then so is $R^k(j)$. But $R^k(j)$ belongs to either S^k_p or S^k_o, which as we have shown earlier contain no profitable deviations. Therefore there can be no profitable deviation in S^k_w.

(b) Principal k's payoff from the $\epsilon-$compromise strategy profile is

$$v^1(0) + v^2(0) - v^{-k}(-k) - \frac{\epsilon}{2} - \frac{\pi}{2} \left(\frac{v^1(1) - v^1(0) + v^2(2) - v^2(0) + \epsilon}{1 - \pi} \right)$$

which is arbitrarily close to $v^1(0) + v^2(0) - v^{-k}(-k)$ for small ϵ and π. Increasing $R^k(0)$ would not change the agent's choice while increasing the reward cost, hence it is not profitable. Lowering $R^k(0)$ would lead the agent to choose either k or $-k$. A switch to k would mean that principal k's payoff is no higher than

$$v^1(0) + v^2(0) - v^{-k}(-k) - \epsilon.$$

With $\epsilon/\pi > v^1(1) - v^1(0) + v^2(2) - v^2(0)$ this would not be profitable either. A switch to $-k$ would of course leave principal k worse off since, by assumption, $v^1(0) + v^2(0) - v^{-k}(-k) > v^k(-k)$. Since lowering $R^k(k)$ would also lead to a switch to $-k$, that would also not be a profitable deviation. Finally, consider raising $R^k(k)$. This would switch the agent's action to k but again bring principal k a payoff no higher than $v^1(0) + v^2(0) - v^{-k}(-k) - \epsilon$, which would again be unprofitable with $\epsilon/\pi > v^1(1) - v^1(0) + v^2(2) - v^2(0)$. Hence in this case ϵ compromise strategies comprise an equilibrium.

Now suppose $v^k(k) + v^{-k}(k) \geq v^k(0) + v^{-k}(0)$ for some $k \in \{1, 2\}$. Then principal $-k$ would do strictly better by setting his reward schedule equal to 0. His payoff would be $v^{-k}(k)$, which is strictly greater than $v^1(0) + v^2(0) - v^k(k) - \frac{\epsilon}{2}$. So the $\epsilon-$compromise strategy profile is not an equilibrium if $v^k(k) + v^{-k}(k) \geq v^k(0) + v^{-k}(0)$.

(c) The main work in proving this part of the theorem is to establish that for small enough π and if either $v^k(k) + v^{-k}(k) > v^k(0) + v^{-k}(0)$ for both $k = 1, 2$ or $a = 0$ is the least preferred outcome for at least one principal, then neither principal would assign positive reward to the action 0, in any equilibrium. Once this is true, it is easy to see that neither principal would reward the other principal's favourite action in any equilibrium. From this point the standard all-pay auction argument delivers the uniqueness result.

To establish that neither principal assigns positive reward to the action 0, in any equilibrium, we go through the following steps. (i) We show that each principal must assign 0 reward to his least favorite action in any equilibrium. (ii) We then show that in equilibrium it cannot be that only a single principal assigns positive reward to the action 0. (i) and (ii) together prove our claim for the case when $a = 0$ is the least preferred outcome for at least one principal. The case in which $a = 0$ is not the least preferred action for either principal is established using the results of part (i) and (ii) in the subsequent part of the proof below.

(i) We first show that each principal must assign 0 reward to his least favorite action in any equilibrium. To see this, denote principal k's least favorite action as $l \in \{0, -k\}$. Consider a strategy $R^k(j)$ with $R^k(l) > 0$. Suppose, given $R^k(j)$ and principal $-k$'s strategy, the agent chooses action k with probability q_k, l with probability q_l and $-l$ with $1 - q_k - q_l$, where $-l \in \{0, -k\}$ such that $-l \neq l$. Principal k's expected payoff is

$$\begin{aligned} q_k &\left(v^k(k) - (1-\pi)R^k(k) - (\pi/2)\left(R^k(l) + R^k(-l)\right)\right) \\ &+ q_l \left(v^k(l) - (1-\pi)R^k(l) - (\pi/2)\left(R^k(k) + R^k(-l)\right)\right) \\ &+ (1 - q_k - q_l)\left(v^k(-l) - (1-\pi)R^k(-l) - (\pi/2)\left(R^k(k) + R^k(l)\right)\right). \end{aligned}$$

For this to be an equilibrium, Principal k's expected payoff must be no less than $v^k(l)$, which he can guarantee by offering no rewards at all. Let $a \in \{k, 0\}$ represent the action that gets positive probability if both q_k and q_{-l} are not positive. If both are positive, then let $a \in \{k, -l\}$ represent the action that generates the higher expected payoff for the principal, the higher $v^k(a) - (1-\pi)R^k(a)$. Let $-a \in \{k, -l\}$ such that $-a \neq a$. For $R^k(j)$ to be played in equilibrium, therefore, we need $v^k(a) - (1-\pi)R^k(a) > v^k(l)$. Consider now the deviation $\widetilde{R}^k(j)$ with $\widetilde{R}^k(a) = R^k(a) + \epsilon$, $\widetilde{R}^k(l) = 0$ and $\widetilde{R}^k(-a) = R^k(-a)$ if

$v^k(a) - (1-\pi)R^k(a) > v^k(l)$ and $\widetilde{R}^k(-a) = 0$ otherwise. This generates an expected payoff of at least

$$\widetilde{q}_a \left(v^k(a) - (1-\pi)\left(R^k(a) + \epsilon\right) - (\pi/2)\left(R^k(-a)\right)\right) \\ + \widetilde{q}_{-a} \max\left\{ v^k(l), \left(v^k(-a) - (1-\pi)R^k(-a) - (\pi/2)\, R^k(a)\right)\right\} \\ + (1 - \widetilde{q}_a - \widetilde{q}_{-a})\left(v^k(l) - (\pi/2)\left(R^k(a) + R^k(-a)\right)\right)$$

where $\widetilde{q}_a \geq q_a$ and $\widetilde{q}_{-a} \geq q_{-a}$. This expected payoff is strictly higher than that from $R^k(j)$, and therefore $R^k(j)$ cannot be played in equilibrium.

(ii) Next, we claim that in equilibrium it cannot be that only a single principal assigns positive reward to the action 0. To see this, consider by contradiction an equilibrium strategy profile in which principal k is the only one to reward action 0 with positive probability. Suppose one such pure strategy is $R^k(j)$. Since $R^k(0) > 0$, by our finding above, 0 cannot be principal k's least favored action, which in turn must be $-k$. So $R^k(-k) = 0$. Suppose, given $R^k(j)$ and principal $-k$'s strategy, the agent chooses action k with probability q_k, 0 with probability q_0 and $-k$ with $1 - q_k - q_0$. Principal k's expected payoff is

$$q_k \left(v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0)\right) + q_0 \left(v^k(0) - (1-\pi)R^k(0) - (\pi/2)\, R^k(k)\right) \\ + (1 - q_k - q_0)\left(v^k(-k) - (\pi/2)\left(R^k(k) + R^k(0)\right)\right).$$

For $R^k(j)$ to be an equilibrium strategy at least one of $v^k(k) - (1-\pi)R^k(k)$ or $v^k(0) - (1-\pi)R^k(0)$ must be strictly greater than $v^k(-k)$, since otherwise switching to no rewards at all would be profitable. Let $a \in \{k, 0\}$ represent the action that gets positive probability if both q_k and q_0 are not positive. If both are positive, then let $a \in \{k, 0\}$ represent the action that generates the higher expected payoff for the principal, the higher $v^k(a) - (1-\pi)R^k(a)$. Now consider a deviation $\widetilde{R}^k(j)$ with $\widetilde{R}^k(k) = R^k(a) + \epsilon$ and $R^k(j) = 0$ for $j \neq k$. This generates an expected payoff of

$$\widetilde{q}\left(v^k(k) - (1-\pi)\left(R^k(a) + \epsilon\right)\right) + (1 - \widetilde{q})\left(v^k(-k) - (\pi/2)\left(R^k(a) + \epsilon\right)\right)$$

where $\widetilde{q} \geq q_k + q_0$, a profitable deviation for small enough ϵ. Since this is a contradiction, we establish that in equilibrium action 0 cannot be rewarded by one principal alone.

So, if 0 is the worst action for at least one principal then neither principal would assign positive reward to the action 0. Then, an almost identical argument to the uniqueness proof of our result in section 3 establishes that the no compromise strategy profile is indeed the unique equilibrium.

Case in which $a = 0$ is not the least preferred action for either principal and $v^k(k) + v^{-k}(k) > v^k(0) + v^{-k}(0)$ for both $k = 1, 2$: To establish that neither principal assigns positive reward to the action 0, in any equilibrium in this case requires more work. From (i) above we know that in this case principal k would never reward action $-k$ and so without loss of generality denote any relevant principal k's strategy as $(R^k(k), R^k(0))$. Now we show that (iii) a pure strategy $(R^k(k), R^k(0))$ where $0 < R^k(k) < R^k(0)$ cannot be played in equilibrium (with positive probability). Furthermore, (iv) if the pure strategy $(R^k(k), R^k(0))$ is played with positive probability then $R^k(0) < \frac{v^k(0)-v^k(-k)}{1-\pi}$. This in turn means that (v) if principal k plays $(R^k(k), R^k(0))$, both strictly positive, in equilibrium, then $R^k(k) - R^k(0) < \frac{v^{-k}(0)-v^{-k}(k)}{1-\pi}$. Then we show that (vi) if $(R^k(k) = r, R^k(0) = p)$ with $0 < p < r$ satisfies (iii)-(v) and principal k is not better off playing $(R^k(k) = r+\epsilon, R^k(0) = 0)$ for small enough ϵ, then principal $-k$ must assign positive probability to strategies of the type $(R^{-k}(-k), R^{-k}(0))$ with $R^{-k}(-k) > R^k(k) = r$ and $R^{-k}(0) + p \geq R^{-k}(-k)$. Our final step (vii) of the argument shows that if $(R^k(k) = r, R^k(0) = p)$ with $0 < p < r$ is played with positive probability in equilibrium then the requirement in step (vi) results in a contradiction.

We start by summarizing the situation. We are assuming that $v^k(k) > v^k(0) > v^k(-k) \geq 0$ with $v^k(k) + v^{-k}(k) > v^k(0) + v^{-k}(0)$ for both $k = 1, 2$. Again we wish to prove that no strategy giving positive reward to action 0 can be played in equilibrium. From our earlier finding we know that only pure strategies of the type $R^k(j)$ where $R^k(-k) = 0$ will get any positive probability. So without loss of generality we can refer to any pure strategy of principal k that can be played in equilibrium as $(R^k(k), R^k(0))$.

(iii) Notice that a pure strategy $(R^k(k), R^k(0))$ where $0 < R^k(k) < R^k(0)$ cannot be played with positive probability. With such a strategy, the agent would never select k, since given that principal $-k$ does not reward k, the total reward on k is strictly less than that for 0. Principal k would do strictly better by deviating either to $(0, R^k(0) + \epsilon)$ for a small enough ϵor to $(0, 0)$. Deviating to $(0, R^k(0) + \epsilon)$ ensures that the agent continues to not take action k and choose 0 with as high a probability as before, if not more. Importantly, principal k saves on the cost of reward following the signal k. Finally, if $(0, R^k(0) + \epsilon)$ is not a profitable deviation for all small ϵ, then it must be that $v^k(-k) \geq v^k(0) - R^k(k)$. But if this were true then $(0, 0)$ constitutes a profitable deviation.

(iv) Next we claim that if the pure strategy $(R^k(k), R^k(0))$ is played with positive probability then $R^k(0) < \frac{v^k(0)-v^k(-k)}{1-\pi}$. Suppose instead that $(R^k(k), R^k(0))$ is played in equilibrium with $R^k(0) \geq \frac{v^k(0)-v^k(-k)}{1-\pi}$. Suppose, given $R^k(j)$ and principal $-k$'s strategy, the agent chooses action k with probability q_k, 0 with probability q_0 and $-k$ with $1 - q_k - q_0$.

Principal k's expected payoff is

$$q_k \left(v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0) \right) + q_0 \left(v^k(0) - (1-\pi)R^k(0) - (\pi/2)\, R^k(k) \right) \\ + (1 - q_k - q_0) \left(v^k(-k) - (\pi/2) \left(R^k(k) + R^k(0) \right) \right)$$

which must be no greater than

$$q_k \left(v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0) \right) + q_0 \left(v^k(-k) - (\pi/2)\, R^k(k) \right) \\ + (1 - q_k - q_0) \left(v^k(-k) - (\pi/2) \left(R^k(k) + R^k(0) \right) \right)$$

If this is less than $v^k(-k)$ then he is better off deviating to $(0,0)$. If not, then it must be that $v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0) > v^k(-k)$. In this case, the principal would be better off deviating to $(R^k(k) + \epsilon, 0)$, for a small enough ϵ, since this generates the strictly higher payoff of

$$\widetilde{q} \left(v^k(k) - (1-\pi) \left(R^k(k) + \epsilon \right) \right) + (1 - \widetilde{q}) \left(v^k(-k) - (\pi/2) \left(R^k(k) + \epsilon \right) \right)$$

with $\widetilde{q} \geq q_k$.

(v) Next we use this to show that if $(R^k(k), R^k(0)) \in S_b^k$ [11]is played with positive probability then

$$R^k(k) - R^k(0) < \frac{v^{-k}(0) - v^{-k}(k)}{1-\pi}.$$

To see this note that we already established that if the pure strategy $(R^{-k}(-k), R^{-k}(0))$ is played with positive probability in equilibrium then $R^{-k}(0) < \frac{v^{-k}(0)-v^{-k}(k)}{1-\pi}$. Now, if principal k were to play $(R^k(k), R^k(0)) \in S_b^k$ with positive probability in equilibrium then he should not profit from deviating to $(R^k(k) + \epsilon, 0)$. Suppose, by contradiction, $R^k(k) - R^k(0) \geq \frac{v^{-k}(0)-v^{-k}(k)}{1-\pi}$. Then $R^k(k) \geq R^k(0) + \frac{v^{-k}(0)-v^{-k}(k)}{1-\pi} > R^k(0) + R^{-k}(0)$. But then for such a strategy profile, the agent would never pick 0. So principal k's expected payoff would be

$$q \left(v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0) \right) + (1-q) \left(v^k(-k) - (\pi/2) \left(R^k(k) + R^k(0) \right) \right)$$

where $q$is the probability with which the agent chooses k. If this were less than $v^k(-k)$, deviating to $(0,0)$would be profitable. If it is higher than $v^k(-k)$ then it must be that

[11] This involves a minor abuse of notation. Recall that $S_b^k = \{R^k(j) | R^k(k), R^k(0) > 0, R^k(-k) = 0\}$. Since we have shown that in the current setting considering $(R^k(k), R^k(0))$ is without loss of generality as in equilibrium $R^k(-k) = 0$, $(R^k(k), R^k(0)) \in S_b^k$ means that $R^k(k), R^k(0) > 0$.

$v^k(k) - (1-\pi)R^k(k) - (\pi/2)\, R^k(0) > v^k(-k)$. In this case, the principal would be better off deviating to $(R^k(k)+\epsilon, 0)$, for a small enough ϵ, since this generates the strictly higher payoff of

$$\widetilde{q}\left(v^k(k) - (1-\pi)\left(R^k(k)+\epsilon\right)\right) + (1-\widetilde{q})\left(v^k(-k) - (\pi/2)\left(R^k(k)+\epsilon\right)\right)$$

with $\widetilde{q} \geq q$.

(vi) Suppose $(R^k(k) = r, R^k(0) = p)$ with $0 < p < r$ satisfies the conditions we have derived so far to be used in equilibrium. We now show that principal k gets a strictly higher payoff with $(R^k(k) = r+\epsilon, R^k(0) = 0)$ than with $(R^k(k) = r, R^k(0) = p)$ for small enough ϵ against any pure strategy used by principal $-k$ *other* than the type $(R^{-k}(-k), R^{-k}(0))$ with $R^{-k}(-k) > R^k(k) = r$ and $R^{-k}(0) + p \geq R^{-k}(-k)$.

To see this, pick any strategy of principal $-k$ which satisfies either $R^{-k}(-k) \leq R^k(k) = r$ or $R^{-k}(0)+p < R^{-k}(-k)$ and consider the change in agent's choice when principal k deviates from $(R^k(k) = r, R^k(0) = p)$ to $(R^k(k) = r+\epsilon, R^k(0) = 0)$. Either the agent's action remains unchanged or it changes from either 0 or $-k$ to k(importantly it cannot change from 0 to $-k$). If the action remains the same then clearly the deviation is a profitable one since it lowers principal k's expected cost. If it changes from 0 to k then the change in principal k's expected payoff would be no less than $\left(v^k(k) - (1-\pi)(r+\epsilon)\right) - \left(v^k(0) - (1-\pi)p\right)$. We know from earlier that for $(R^k(k) = r, R^k(0) = p)$ to be played with positive probability in equilibrium it must be that $r - p \leq \frac{v^{-k}(0) - v^{-k}(k)}{1-\pi}$. So the change in principal k's expected payoff would be no less than $v^k(k) + v^{-k}(k) - v^k(0) + v^{-k}(0) - (1-\pi)\,\epsilon$. For small enough ϵ this must be positive since by assumption $v^k(k) + v^{-k}(k) > v^k(0) + v^{-k}(0)$. Therefore, in this case too, the deviation would be profitable. Finally consider the agent's choice changing from $-k$ to k. The change in principal k's expected payoff would be strictly higher than $\left(v^k(k) - (1-\pi)(r+\epsilon)\right) - v^k(-k)$. If this is positive the deviation is profitable. If this is negative then $(R^k(k) = r, R^k(0) = p)$ cannot be played with positive probability in equilibrium, since principal k would be better off playing $(0,0)$. So we conclude that for $(R^k(k) = r, R^k(0) = p)$ with $0 < p < r$ to be used by principal k in equilibrium, principal $-k$ must put positive probability on a strategy$(R^{-k}(-k), R^{-k}(0))$ with $R^{-k}(-k) > R^k(k) = r$ and $R^{-k}(0) + p \geq R^{-k}(-k)$.

(vii) We now complete the proof. For a given equilibrium strategy profile, suppose $\widetilde{S}_b^k$ consist of all the strategies in S_b^k that get positive probability. Let $\overline{R}^k(k) = \sup\left\{R^k(k) \mid (R^k(k), R^k(0)) \in \widetilde{S}_b^k\right\}$. If $\widetilde{S}_b^k$ is non-empty then by the argument of the paragraph above it must be that $\bar{R}^1(1) = \bar{R}^2(2)$. Further it cannot be that there is a probability mass on some $(R^k(k), R^k(0)) \in \widetilde{S}_b^k$ with $R^k(k) = \overline{R}^k(k)$. But then for principal k to not profitably deviate to $(\bar{R}^k(k), 0)$, principal $-k$ must have a probability mass on a strategy $(R^{-k}(-k), R^{-k}(0)) \in \widetilde{S}_b^{-k}$ with

$R^{-k}(-k) = \overline{R}^{-k}(-k)$.. This is a contradiction. So it must be that $\widetilde{S}_b^k$ is empty, meaning strategies in S_b^k are not played with positive probability in equilibrium. Therefore in equilibrium each principal must only be assigning rewards to their favorite action. Then, an almost identical argument to the uniqueness proof of our result in section 3 establishes that the no compromise strategy profile is indeed the unique equilibrium. □