1. Commitment
   Demand is given by $p = 17 - x$. Firm 1, the Stackelberg leader chooses output, moves first and has a marginal cost of 3. Firm 2, the Stackelberg follower, chooses output, moves second and has a marginal cost of 1. Find the Stackelberg equilibrium of this game, which is the same as the subgame perfect equilibrium of the Stackelberg game, including both the output and profits of both firms, and the equilibrium market price.
   $\pi_1 = px_1 - 3x_1 = 17x_1 - x_1^2 - x_1x_2 - 3x_1$
   $\pi_2 = 17x_2 - x_2^2 - x_1x_2 - x_2$
   Suppose Firm 1 chooses an output level x_1.
   Firm 2, to optimize profits, must produce x_2 such that,
   $17 - 2x_2 - x_1 - 1 = 0$ (from the first order condition)
   So $x_2 = \frac{16-x_1}{2}$
   By backward induction, Firm 1 must choose output level x_1 given that Firm 2 will produce $x_2 = \frac{16-x_1}{2}$
   So the profit function for Firm 1 becomes,
   $\pi_1 = px_1 - 3x_1 = 17x_1 - x_1^2 - x_1\frac{16-x_1}{2} - 3x_1 = 6x_1 - \frac{x_1^2}{2}$
   Maximizing this function requires, $x_1 = 6$
   So the Subgame Perfect Equilibrium of this game involves $x_1 = 6$ and $x_2 = 5$.
   $\pi_1 = 18$, $\pi_2 = 25$
   The equilibrium market price is $p = 6$.

2. Save the whales!
   Consider the following symmetric game in which two countries decide whether or not to pollute, or whether to pollute heavily. Suppose that it is infinitely repeated with discount factor $0 < d < 1$.
   a) Give an accurate sketch of the socially feasible individually rational set. What does the Folk Theorem say about this set?
   The Folk Theorem states that any element of this set (except possibly the boundaries) can be supported as a Subgame Perfect Equilibrium for d close enough to 1.

   b) Find grim trigger strategies so that players both get 0 in equilibrium. For what discount factors are these a Nash equilibrium?
   The grim trigger strategies would involve playing (Not Pollute) as long as in every earlier period (Not Pollute, Not Pollute) has been played. If anything else has been played in the past, keep playing (Pollute) forever. For these strategies to constitute an SPE it must be that the payoff from deviating, $(1-d)1 + d(-1)$ should be less than the equilibrium payoff of 0. So $d > \frac{1}{2}$

   c) Explain why the strategies derived above are subgame perfect?
   If $d > \frac{1}{2}$, as mentioned in the argument above, no player can gain by playing (Pollute) when they are required to play (Not Pollute). In the subgames where the players are required to (Pollute) they are playing the

static Nash equilibrium and hence cannot gain in the present period by deviating. Further since in these subgames, the strategy in subsequent rounds is independent of what is played in the present period , they cannot improve their future payoffs either by deviating.
d) Explain the difference between the "full" folk theorem involving individual rationality and the "Nash threats" folk theorem in this game. What is the static Nash equilibrium?
The "Nash threats" folk theorem in this game would suggest that only those payoffs in the socially feasible individually rational set where each player gets at least her lowest Static Nash payoff can be supported by an SPE. The full folk theorem states that any element (except for some of the boundaries) of the socially feasible individually rational set can be supported. The difference arises because the former restricts the set of possible threats to be Static Nash Equilibria themselves, while the full folk theorem does not make such a restriction.
The static Nash Equilibrium is (Pollute, Pollute).

3. Long Run versus Short Run
A short-lived politician must choose whether to pass honest legislation, yielding him a payoff of 1, and to the long-run taxpayer of 1, or whether to take a bribe. If the politician takes the bribe, the taxpayer must decide whether or not to investigate. If the taxpayer does not investigate the matter, the politician gets 2, but the taxpayer gets -1 on account of the dishonest legislation. If the taxpayer investigates, then the politician gets -1, but the investigation is expensive and government services are disrupted, so the taxpayer gets -2. If the politician passes an honest legislation, investigation results in the taxpayer getting 1, while not making such an attempt, the taxpayer gets 2.
a) Find the extensive and normal form of this game.

b) What pure strategy Nash equilibria are there in the stage game; which are subgame perfect? What is the Stackelberg equilibrium of the stage game in which the taxpayer moves first?
Pure Strategy Nash Equilibria : (Not Investigate, Bribe)
Subgame Perfect: (Not Investigate, Bribe)
Stackelberg Equilibrium in which the taxpayer moves first: (Investigate, Honest)

c) If the stage game is repeated and the taxpayer is infinitely lived with discount factor equal to d and there is a sequence of short-lived politicians propose a subgame perfect equilibrium strategy and a d such that players end up playing Stackelberg every period.

Strategy for Taxpayer: Investigate in the first period and continue playing

Investigate as long as Not Investigate or Bribe has not been played in the past.
If either Not Investigate or Bribe has been played in the past play Not Investigate.

Strategy for Politician: Play Honest in the first period and continue playing Honest as long as Not Investigate or Bribe has not been played in the past.
If either Not Investigate or Bribe has been played in the past play Bribe.
The equilibrium outcome given these strategies would be (Investigate, Honest) in every period.
For this to be a subgame perfect equilibrium, the taxpayer should not be able to benefit by deviating.
Payoff from deviating is $(1-d)2 + d(-1)$
Payoff from not deviating is 1
So for $1 > 2 - 3d$, it must be that $d > \frac{1}{3}$
The politician is always playing the static best response to the taxpayers strategy and hence can't do better by deviating.
Note: Playing a Nash Equilibrium (of the stage game) in every period is Subgame Perfect, since the stage game is a simultaneous move game, having no subgames other than the game itself.