1. Suppose that there are two good, the quantities of which are denoted by x, y , and a consumer with utility given by $x + 2y - y^2$. The consumer has income I and the prices are p, q .
   a. Write the budget constraint of the consumer.
   $px + qy \leq I$

   b. Find the demand of the consumer for x as a function of p, q, I . From the first order conditions we get, $q = 2(1-y)p$. So $y = 1 - \frac{q}{2p}$. Substituting in the budget constraint gives,
   $px + q(1 - \frac{q}{2p}) = I \Rightarrow x = \frac{I - q(1-\frac{q}{2p})}{p}$.
   So if $2p < q$ the demand for x is $\frac{I}{p}$.
   If $2p > q$ the demand for x is $max\{\frac{I-q(1-\frac{q}{2p})}{p}, 0\}$
   Suppose that there is a firm that produces the good x at constant marginal cost c .

   c. Find the competitive equilibrium output and price in the market for x . Firstly, it must be that $p = c$
   If $2c > q$ and $I > q(1 - \frac{q}{2c})$ then the demand for x is $\frac{I-q(1-\frac{q}{2c})}{c}$.
   If $2c < q$ then $x = \frac{I}{c}$
   Else $x = 0$

   d. What happens to the revenue of a monopolist as it changes price in the market for good x ? In light of what happens with revenue, explain in general terms what the monopolist should do.
   The revenue for the monopolist would be $R = x.p$.
   If $2p < q$, $R = I$
   If $2p > q$, $R = I - q(1 - \frac{q}{2p}) < I$
   So if the monopolist wishes to maximize revenue she should set $p = \frac{q}{2}$. In general however, the price chosen would depend upon the cost function for the monopolist.

2. In the following simultaneous move matrix game, find the dominant strategy equilibrium (if any), apply iterated strict dominance to reduce the size of the game, find the reaction functions of the two players (in the original game, not the reduced game), and find as many Nash equilibria as you can. Is there any Nash equilibrium that is Pareto efficient?

   There is no dominant strategy equilibrium.
   Iterated strict dominance removes the column R and row D.
   The reaction function for player 1 is $R^1(L) = U$, $R^1(C) = M$ and $R^1(R) = U$.
   For player 2 it is $R^2(U) = L$, $R^2(M) = C$ and $R^2(D) = L$.
   The two pure strategy Nash Equilibrium are (U, L) and (M, C).

(U, L) is Pareto efficient.

3. Suppose that two firms $i = 1, 2$ each produce output x_i and the profit of firm i is $x_i - x_i^2 - x_i x_{-i}$. Find the Cournot (same meaning as Nash) equilibrium in which both firms choose quantities.

   Player 1's reaction function is $R^1(x_2) = \frac{1-x_2}{2}$.
   Player 2's reaction function is $R^2(x_1) = \frac{1-x_1}{2}$.
   Therefore the Cournot equilibrium (x_1^*, x_2^*) must satisfy,
   $x_1^* = \frac{1-x_2^*}{2} = \frac{1-\frac{1-x_1^*}{2}}{2}$
   This simplifies to $x_1^* = \frac{1}{3}$.
   Similarly, $x_2^* = \frac{1}{3}$